Cyclical variations in liquidity risk of corporate bonds

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Abstract

We study regime switching features of liquidity risk in corporate bond premiums. Within a sample period ranging from July 2002 to April 2015, we first compute a liquidity risk index for BBB bonds, which considers various liquidity risk facets based on principal component analysis. Second, we identify two liquidity regimes in our sample using a Markov switching regime model that highlights the dynamic characteristics of this risk and its behavior before, during and after the last financial crisis. We observe that the liquidity risk index improved after the financial crisis. It seems that the recent Volcker Rule did not affect the liquidity of BBB bonds during our sample period.

Keywords: Liquidity risk, corporate bonds, financial crisis, regime change, Markov model, principal component analysis.

1. Introduction

The recent financial crisis of 2007-2009 was one of the most consequential events of the past decade in the financial world. The ensuing subprime crisis was caused by various events linked to that phenomenon. The year 2007 was marked by a crisis in asset-backed commercial paper (ABCP), along with the sharp downgrading of collateralized debt obligations (CDOs). Subsequently, several large financial institutions sustained serious damage in 2008, including the collapse of Bear Stearns in March, problems at IndyMac, Fannie Mae and Freddie Mae, and the Lehman Brothers bankruptcy in September. Other financial institutions like AIG had to be rescued by the US Government to avert a possible collapse of the financial system. In October 2008, the American government signed an asset rescue program called the Troubled Asset Relief Program (TARP). This program consisted of buying risky assets from financial institutions in order to inject liquidity in the economy.

Many observers claim that the recent financial crisis had two components: 1) a default crisis, and 2) a liquidity crisis, notably on the bond market. The magnitude of the latter crisis was almost unprecedented. Saunders and Allen (2010) identify the liquidity crisis as extending from July 2007 to August 2008, and the default crisis from September 2008 to March 2009. Dionne and Maalaoui Chun (2013) show that the liquidity crisis began in July 2007 and ended in March 2009, and the default crisis covered June 2008 to January 2009 (Table 1).

During the recent crisis, liquidity risk reached record levels. The US government's decision to implement the TARP illustrates the gravity of the situation. International financial regulators thus decided to focus on liquidity risk, previously a minor concern. The result was the publication of the Basel III Accords in 2010. The implementation of these accords continues until 2019, and is designed to improve management of liquidity risk of financial institutions, notably by defining the regulatory capital that banks must set aside to hedge this risk. In addition, the Dodd-Frank Act was implemented in the United States in 2010. Its main objective was to reduce banks' speculative activities. It may have generated side effects on bonds' liquidity. However, liquidity risk is not yet well defined, and very few studies have examined its cyclical evolution.

In this research, we are particularly interested in liquidity risk present in the US bond market. We gather data from TRACE (*Trade Reporting And Compliance Engine*) to obtain information on intraday bond trading on the US market during the July 2002 to April 2015 period. We concentrate our analysis on the BBB corporate bond market.

In the first step, we apply the methodology of Dick-Nielsen et al. (2012) to measure this risk by performing a principal component analysis (CPA) on eight predetermined measures. Second, we seek to identify the properties of liquidity risk present in bonds by analyzing the dynamic behavior of this risk in our sample, using a Markov switching regime model. Our main motivation for the use of a Markov model is to analyze the cycles of this risk. If the liquidity risk cycles are important, their presence may modify the calculation of the average capital of financial institutions over a period of several cycles, as has been demonstrated for default and operational risk (Chen, 2010; Bhamra et al., 2010; Dionne and Saissi Hassani, 2017).

The paper is organized as follows. Section 2 presents a review of the literature on liquidity risk in bond premiums. Section 3 specifies the details of the data used in our study. We then build our illiquidity index. Section 5 discusses the methodology of the regime change model that lets us identify the liquidity regimes within our sample. Lastly, we analyze the results obtained to determine whether or not cyclical variation of liquidity risk must be taken into account when managing optimal capital.

2. Literature

2.1 Context

The emergence of liquidity risk in the last financial crisis has underlined the importance of better understanding this risk. This risk prompted the changes to the risk management regulation of banks in 2010. Specifically, under Basel III, regulated banks must now calculate two ratios to demonstrate their protection from liquidity risk:

- The Liquidity Coverage Ratio (LCR)
- The Net Stable Funding Ratio (NSFR)

The objective of the LCR is to allow financial institutions to prove that they can withstand liquidity risk. It is a means for the regulators to ensure that banks have enough highly liquid assets to face a stress scenario for one month. The NSFR encourages banks to prove their long-term robustness by using more stable sources of financing in their operations (Gomes and Khan, 2011). Banks are asked to set aside more capital, especially better quality capital. In addition, the capital standards are more flexible and the requirements also concern banks' leverage (Gauthier and Tomura, 2011; Bao et al., 2016; Anderson and Stulz, 2017).

Our study period covers the introduction of the Dodd-Frank Act in the United States. The Act includes the Volcker Rule, designed to reduce banks' speculative activities. According to Duffie (2012), the Rule could have side effects on bond liquidity that are attributable to a reduction in banks' market activities. Recent studies of bond liquidity after the financial crisis diverge regarding the evolution of bond liquidity risk (Trebbi and Xiao, 2016; Dick-Nielsen et al., 2012; Bessembinder et al., 2016; Bao et al., 2016). Given that our data period partly covers the post-crisis period, we will see how the Volcker Rule affected the cyclical variation of BBB liquidity risk.[1]

We concentrate our study on liquidity risk in bond premiums because it is possible to access high-quality data on this asset class, and because the risk of these financial assets was relatively important during the last financial crisis (Dick-Nielsen et al., 2012). In fact, because bonds are not traded on an exchange but rather on the over-the-counter market, information on these transactions has been very opaque for many years. However, since January 2001, members of the *Financial Industry Regulatory Authority* (FINRA) have been required to report their secondary market of over-the-counter bond transactions to the TRACE database. This has increased transparency on the bond market since the launch of

[1] These studies are discussed in the articles by Bao et al. (2016) and Anderson and Stulz (2017). Anderson and Stulz (2017) find a drop in bond turnover after the financial crisis explained by the trading of smaller quantities, yet traders continued to trade as often as in the period preceding the financial crisis. We will see that bond turnover explains only 12.5% of BBB bond total variance.

TRACE on July 1, 2002, which has shed more light on this financial asset traditionally known for its illiquidity.

2.2 Credit spread puzzle

The study by Huang and Huang (2012, first version in 2002) traces the origin of the *credit spread puzzle*, namely that structural models of default can explain only a portion of the bond premiums of bond yield spreads. Bond premiums are therefore no longer based on default risk alone. Huang and Huang (2012) found a proportion of default risk of about 20% for top-quality bonds.

Eom, Helwege and Huang (2004) tested five default structural models and concluded that such models do not explain total bond spreads. They find that the five studied structural models underestimate the total spread of the least risky bonds, namely those issued by firms that have little leverage or asset volatility, and they observe an overestimation of total spreads in the riskiest firms.

Elton et al. (2001) also sought to explain the composition of bond premiums using a statistical model. Their research rests on the premise that bond premiums are explained by an unexpected default risk, a tax premium and a market risk premium. To determine the portion of the spread due to default risk, the authors use a model where the marginal default probabilities are computed using a transition matrix. With a sample ranging from 1987 to 1996, the authors find that default risk explains only about 25% of bond spreads, a result similar to that obtained by Huang and Huang (2012) and Collin-Dufresne et al. (2001). Dionne et al. (2010) report a higher proportion for default risk (about 50%) by considering different transition matrices from 1987 to 1996.

Given the questions raised by the credit spread puzzle, new techniques were developed to estimate the default risk inherent in bond premiums. Longstaff et al. (2005) explain the composition of bond spread by using CDS premiums to measure default risk. Their main hypothesis is that CDS premiums are composed solely of the default risk of bonds on which the contract is written. CDS buyers want a contract to protect themselves from a possible default by the bond-issuing firm. In exchange for payments of quarterly premiums to CDS sellers, buyers obtain the face value of the bond if the bond issuer defaults before the expiry of the hedging contract. With a sample ranging from 2001 to 2002, Longstaff et al.'s (2005) results imply that default risk represents about 53% of premiums of superior quality bonds, and that the lower the bond's credit rating, the greater the proportion of default risk in the spread. They assert that default risk represents most of the bond spread for different credit ratings, contrary to previous studies. However, their sample covers only a short period with few observations. Further, Longstaff et al. (2005) studied the non-default component of yield spreads. They found little evidence that this component is influenced by taxes, in contrast with the study by Elton et al. (2001). In addition, they affirm that the non-default portion is linked to illiquidity measures specific to bonds and to macroeconomic measures of bond market liquidity. Lin, Wang and Wu (2011) and Chen, Lesmond and Wei (2007) also confirm a non-negligible presence of liquidity risk in bond premiums.

2.3 Liquidity risk in bond premiums

Different types of modelling of liquidity risk have been developed in response to the credit spread puzzle. It is important to identify the factors that may determine the magnitude of this risk. Some studies assert that the differences found between higher-quality versus lower-quality bonds can be explained by macroeconomic factors. Also, a flight to quality was observed during the financial crisis for high-quality bonds that are less subject to liquidity risk (Dick-Nielsen et al., 2012).

Han and Zhou (2008) analyze the effect of liquidity on the non-default component of bond yields. Their study builds on that of Longstaff et al. (2005), although they specifically assess the importance of liquidity risk in bond spreads. To measure the default component, Han and Zhou (2008) also use CDS premiums. They supplement their analysis with different measures of liquidity risk. Notably, after doing regressions on all of these measures and by controlling for other factors, the authors find a link between liquidity and the non-default component of higher-quality bonds, but not for speculative bonds. In addition, the non-default component seems to be greater for BBB bonds. It may reach up to 50% of the total spread. Bao, Pan and Wang (2011) observed that liquidity risk played a greater role in explaining temporal variations in bond premiums than did default risk in the case of higher-quality bonds (ratings between AAA and A) compared with other bonds.

A link between liquidity risk and the macroeconomic environment was observed by Dionne and Maalaoui Chun (2013), who, using a regime shift detection model, established a correspondence between the persistence of bond spreads and their power to predict economic cycles. Specifically, they affirmed that the liquidity regime has predictive power over bonds' total yield spread, whereas default risk seems to be more associated with the persistent nature of the spread. They conclude that the last financial crisis began with an increase in liquidity risk in bond premiums followed by a persistent increase in default risk. In this study, we use a Markov regime switching model instead of a regime shift detection model. To our knowledge, this is the first time that the Markov model is applied to liquidity risk.

Dick-Nielsen, Feldhütter and Lando (2012) observed an increase in the liquidity risk component of premiums of all bonds except for AAA bonds, at the start of the last financial crisis. Whereas they observed an increase of 5 basis points in AAA bond premiums, liquidity rose by 93 basis points for BBB bonds and between 58 and 197 basis points for speculative bonds. Friewald, Jankowitsch and Subrahmanyam (2012) obtained similar results concerning this flight-to-quality phenomenon. With a sample ranging from October 2004 to December 2008, they analyzed two crisis periods: the General Motors (GM)/Ford crisis and the subprime crisis. Their results are interesting in that the comparison of the two crises shows that the flight to quality seems to be more evident during the subprime crisis than the GM/Ford crisis. Accordingly, they observe a decline in the number of bonds traded and in transactions on speculative bonds during the subprime crisis, whereas for higher-quality bonds the number of bonds traded remained the same, while the number of transactions increased. This clearly reflects the flight-to-quality phenomenon that occurred during this crisis. However, for the GM/Ford crisis, the authors observed, in contrast, an increase in speculative bond trading. Finally, Maalaoui Chun et al. (2014) broke down bond spreads into three important components: default risk, liquidity risk and general market risk. They concluded that liquidity risk represents about 20% of BBB bond spreads whereas the default risk factor represents 18% and the market risk factor 28%.

3. Data

We use the intraday data of US BBB corporate bonds in a sample that extends from July 1, 2002 to March 31, 2015. The data come from TRACE. The Dick-Nielsen (2009) filter lets us eliminate transactions between clients and brokers, which are agency transactions. We use the ratings provided by Moody's to identify BBB bonds. When a bond changes rating and is no longer BBB, it is removed from our panel. Conversely, if it becomes a BBB bond we add it to the sample. We thus do not keep bonds in periods of stress, as Bao et al. (2016) did.

Among the bonds rated BBB, we keep straight bonds. These bonds possess no options such as redemption, conversion or sinking fund provision. In addition, the bonds selected distribute only fixed coupons and are senior and unsecured. We used FISD (Fixed Income Securities Database) data to select the BBB bonds. After having applied these filters, we obtained a sample of 7,229 US corporate BBB bonds and 3,607,635 intra-day prices between July 1, 2002 and March 31, 2015.

4. Construction of the illiquidity risk index

4.1 Measures used

To obtain our illiquidity risk index, we begin by calculating the eight measures of liquidity risk specified by Dick-Nielsen et al. (2012). Then we apply a PCA to quantify an index of illiquidity. This approach lets us capture different facets of liquidity risk inherent in bond premiums and retain the essential components in a single measure.

<u>Amihud</u>

Amihud's measure (2002) is one of the most widely used measures of financial product illiquidity. Its initial purpose was to measure illiquidity in stock returns, but, as Dick-Nielsen et al. (2012) and Dionne and Maalaoui Chun (2013) demonstrated, it can also be used for other financial products such as bonds. To affirm the hypothesis that bond returns must increase with illiquidity, we create a measure defined as the mean ratio of absolute daily return on volume in dollars traded that day. This ratio thus gives the daily impact on bond prices of one dollar of volume traded. For each bond *i*, we calculate the Amihud daily measure such that:

$$Amihud_t^i = \frac{1}{N_t^i} \sum_{j=1}^{N_t^i} \frac{1}{Q_{j,t}^i} \frac{\left| P_{j,t}^i - P_{j-1,t}^i \right|}{P_{j-1,t}^i} \tag{1}$$

where:

N_t^i represents the number of transactions of bond *i* observed on day *t*;

$P_{j,t}^i$ corresponds to the price of the j^{th} transaction of bond *i* on day *t*;

$Q_{j,t}^i$ (in dollars) represents the j^{th} trading volume of bond *i* on day *t*.

To calculate this measure, we apply the filter of Han and Zhou (2008), as in the study by Dionne and Maalaoui Chun (2013). This consists in excluding the price of transactions below $1 and over $500, along with prices that are 20% higher than the median price of the same day or the previous day. To obtain the measure we must verify that we have at least three transactions for each bond during a given day.

Roll

Roll (1984) developed a measure of effective bond spread. He pointed out that obtaining the bid-ask spread is difficult because these data are rarely published, and measurement of transaction costs may lead to different types of errors due to the various factors that must be considered. He therefore proposes an easy and inexpensive way to approximate this valuable information. Note that since November 2008 it is possible to determine whether a transaction price concerns a purchase or a sale. Given that our sample ranges from July 2002 to April 2015 we use the Roll measure to ensure uniformity throughout our period of analysis:

$$Roll_t^i = 2\sqrt{-cov(\Delta P_t^i, \Delta P_{t-1}^i)} \tag{2}$$

where ΔP refers to changes in transaction prices around a mean. Accordingly, the two variations in price are of opposite signs, generating negative covariance. This measure is calculated using a rolling window of 21 trading days that include at least four transactions.

IRC (*Imputed Roundtrip Cost*)

An Imputed Roundtrip Cost can be defined as the difference between the price at which a dealer sells a bond to a client and the price at which the dealer buys the bond from another client. Feldhütter (2012) builds on this notion by introducing the IRT (Imputed Roundtrip Trade). In the IRT, the highest price (max) corresponds to the investor's purchases from the dealer, and the lowest price (min) is the price at which the investor sells to the dealer. The investor's Imputed Roundtrip Cost (IRC) is the difference between these two prices. Given that the IRT reflects transaction costs, we can expect that the higher the IRT, the more illiquid the market. Therefore for each day, we calculate the mean estimates of daily IRC for different trading volumes. The measure is expressed as follows, with P^i_{max} and P^i_{min} representing the maximum and minimum price in the round-trip cost for the same volume:

$$IRC^i_t = \frac{P^i_{max} - P^i_{min}}{P^i_{max}} \tag{3}$$

This is a pure price index of illiquidity, as it is for the Roll measure.

Turnover

The Turnover measure lets us calculate the daily turnover of a bond. The inverse of this measure indicates the average holding time of a bond. The longer a bond is held, the greater the inverse of the Turnover and the less liquid the bond.

$$Turnover^i_t = \frac{total\ trading\ volume^i_t}{amount\ outstanding^i} \tag{4}$$

Zero bond

Lesmond, Ogden and Trzcinka (1999) analyze another measure of illiquidity based on transaction costs: the zero-return measure. They thus propose an alternative to the usual yield spread by using time series of daily returns of a financial asset. This measure is the number of zero trading days for a given asset during a given period. If it costs too much for dealers to carry out a transaction by considering the value of their information signal, they will reduce their trading or not trade at all, which would translate into a zero return.

Therefore, the higher the number of zero trades during a period, the more illiquid the bond. We obtain the zero bond measure by using a rolling window of 21 trading days:

$$zero\ bond_t^i = \frac{\text{number of zero bond } i \text{ trades within the rolling window}}{\text{number of days in the rolling window}}. \tag{5}$$

Zero firm

The zero firm measure is very similar to the zero bond measure. It measures the number of days in which none of a firm's bond issues is traded. As above, we obtain a measure for bond *i* of firm *j* by using a rolling window of 21 trading days:

$$zero\ firm_t^{ij} = \frac{\text{number of zero firm } j \text{ trades within the rolling window}}{\text{number of days in the rolling window}} \tag{6}$$

Amihud Risk and IRC Risk

The daily measures Amihud Risk and IRC Risk refer to the standard deviation of the Amihud and IRC measure respectively. These two measures thus capture the variability in bond illiquidity. They are constructed using a rolling window of 21 trading days.

4.2 Liquidity risk index

After calculating the various measures, we can perform a principal component analysis (PCA) to obtain a liquidity risk index. Our illiquidity index therefore corresponds to a linear combination of the results obtained from the principal component analysis. Specifically, each measure selected by the PCA is defined by l_{it}^k, where *i* is for a bond, *t* is a measure of time, and *k* corresponds to a chosen measure of liquidity risk. Each measure is then standardized to obtain $\bar{l}_{it}^k = \frac{l_{it}^k - \mu_k}{\sigma_k}$, where μ_k and σ_k correspond to the mean and standard deviation of the liquidity risk measure *k*. The daily measure of liquidity risk φ_{it} is then obtained by a linear combination of different standardized measures of liquidity:

$$\varphi_{it} = \sum_{k=1}^{K} \lambda^k\, \bar{l}_{it}^k \tag{7}$$

where K is the number of liquidity measures retained from the principal component analysis and λ^krepresents their different weights.

5. Identification of liquidity regimes

We assume that the data are part of a dynamic system of two possible states. At any time, the system can be in any state. Our objective is to distinguish high liquidity risk periods from low liquidity risk periods. We use information on the illiquidity index identified in the previous section exclusively. In Appendix A.1, we describe the Markov model in detail.

We use a daily time series of our measure of liquidity risk. We estimate a dynamic regression model that best corresponds to daily data frequency. Our model can thus be described as follows:

$$\text{State 1: } Rliq_t = \mu_1 + \varepsilon_1$$

$$\text{State 2: } Rliq_t = \mu_2 + \varepsilon_2$$

with $Rliq_t$ as the liquidity risk measure.

The transition probabilities p_{11}, p_{12}, p_{21} and p_{22} are estimated. Estimating the transition probabilities of our model leads us to calculate the ergodic probabilities of a two-state Markov chain. Specifically, we can detect an ergodic Markov chain.

To better represent the two regimes identified in our sample, we will calculate the values predicted by our model. Note that in our model, μ_1 represents the mean of our measure of liquidity risk during regime 1, and μ_2 the mean during regime 2. We can then observe how our measure behaves over time, relative to economic events such as the last financial crisis, which ran from July 2007 to March 2009, and the last NBER recession, which ran from December 2007 until June 2009.

6. Analysis of results

6.1 Principal component analysis (PCA)

The goal of a PCA is to transform a set of potential correlated variables into a set of linearly independent variables or principal components (PC). The first PC obtained is the component that explains most of the total illiquidity variance. This procedure is particularly useful when we suspect strong correlation between the variables that we plan to use. Moreover, the methodology helps reduce model overfitting.

To create our illiquidity risk index, we perform a principal component analysis that uses as inputs the eight measures proposed in Section 4. Given that these measures capture different facets of liquidity risk, we want to obtain a global measure that would represent a synthesis of the previous measures. First, we review the statistics for these measures in Tables 1 and 2. After having applied the filter of Han and Zhou (2008), we retained 2,808,041 prices, which represent about 78% of our initial sample. This filter excludes irregularities in bond transaction prices.

	Amihud	Amihud Risk	IRC	IRC Risk	Roll	Turnover	Zero Bond	Zero Firm
Mean	0.8461	0.9116	0.0073	0.0054	1.4692	0.2406	0.2293	0.0780
Standard deviation	1.8562	1.4977	0.0081	0.0043	1.2185	23.5365	0.2461	0.1761

Table 1: Means and standard deviations of eight standardized illiquidity measures

Table 1 indicates that the standardized measures with the highest standard deviations are Turnover, at 23.54, followed by the Amihud, Amihud Risk and Roll measures, at 1.86, 1.50 and 1.22 respectively. Turnover is a liquidity measure, unlike the other illiquidity measures. Further, the measures with the lowest standard deviations are IRC and IRC Risk. These values are not directly comparable but may be useful when we look at the results of the PCA.

	IRC Risk	Amihud Risk	Zero Bond	Turnover	Roll	Amihud	IRC	Zero Firm
IRC Risk	1	0.3590	0.0557	0.0034	0.5254	0.2394	0.4594	0.0293
Amihud Risk	0.3590	1	0.0307	0.0008	0.3114	0.4527	0.2007	0.0198
Zero Bond	0.0557	0.0307	1	0.0148	0.0166	0.0297	0.0419	0.3945
Turnover	0.0034	0.0008	0.0148	1	0.0046	-0.0014	0.0004	0.0063
Roll	0.5254	0.3114	0.0166	0.0046	1	0.2098	0.3296	0.0139
Amihud	0.2394	0.4527	0.0297	-0.0014	0.2098	1	0.3432	0.0138
IRC	0.4594	0.2007	0.0419	0.0004	0.3296	0.3432	1	-0.0023
Zero Firm	0.0293	0.0198	0.3945	0.0063	0.0139	0.0138	-0.0023	1

Table 2: Correlation matrix of eight illiquidity measures

Table 2 shows that some measures seem to have a high correlation with the others. The measures with the strongest correlations are IRC Risk, Amihud Risk, Roll, Amihud and IRC. The Roll and IRC Risk measures demonstrate the strongest correlation, at 0.53. The IRC measure has a correlation of 0.46 with IRC Risk, followed closely behind by Amihud and Amihud Risk with 0.45. These correlations justify the use of a PCA.

Table 3 shows the eigenvalues of the PCA, which indicate the percentage of total variance explained by each eigenvector chosen, whose details are presented in Table 4. Eight eigenvalues are calculated, and the proportion of the total variance explained in the last column depends on the eigenvector or principal component chosen. For example, the first eigenvector (PC1) explains 29.84% of the variance of our sample. By comparison, the second eigenvector (PC2) explains 17.37% of the total variance. The third principal component (PC3) explains 12.5% of the total illiquidity variance.

	Eigenvalue	Difference	Proportion	Cumulative
PC1	2.38726873	0.99799498	0.2984	0.2984
PC2	1.38927375	0.38937574	0.1737	0.4721
PC3	0.99989801	0.04106077	0.1250	0.5971
PC4	0.95883724	0.19952469	0.1199	0.7169
PC5	0.75931255	0.15594887	0.0949	0.8118
PC6	0.60336368	0.10195510	0.0754	0.8872
PC7	0.50140857	0.10077111	0.0627	0.9499
PC8	0.40063746		0.0501	1

Table 3: Eigenvalues of the eight PCA measures

Table 4 illustrates the composition of the different eigenvectors. All of the measures of illiquidity are represented therein. We can also observe their contributions to the eigenvectors, which lets us choose the measures to incorporate in our liquidity risk index.

	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8
IRC Risk	0.5004	-0.0189	0.0335	-0.3758	-0.1124	-0.0396	-0.4127	-0.6499
Amihud Risk	0.4284	-0.032	-0.0418	0.4955	-0.4561	-0.1019	-0.4334	0.4024
Zero Bond	0.0695	0.7020	-0.0155	-0.008	0.0924	-0.6973	0.0848	0.0096
Turnover	0.0036	0.0373	0.995	0.0915	0.0079	0.0154	-0.0029	-0.0004
Roll	0.4520	-0.0518	0.0422	-0.4033	-0.4025	0.0240	0.6353	0.2497

Amihud	0.4046	-0.0364	-0.0603	0.6278	0.2865	0.0844	0.4431	-0.3895
IRC	0.4372	-0.0432	0.0128	-0.2193	0.7205	0.0823	-0.1776	0.4486
Zero Firm	0.0443	0.7061	-0.0374	0.0049	-0.0866	0.698	-0.0564	0.0171

Table 4: Eigenvector composition

From Tables 3 and 4, we conclude that the chosen illiquidity risk index is made up of the following five measures: IRC Risk, Roll, IRC, Amihud Risk and Amihud. These measures present the strongest coefficients in PC1, at 0.50, 0.45, 0.44, 0.43, and 0.40 respectively, and they explain almost 30% of the total variance. Note that the results observed with the correlation matrix in Table 2 already suggested this output because we previously noted that these measures had the strongest correlations. Given that the five coefficients are fairly similar, we opt to give them equivalent weight in constructing the index. In addition, no eigenvector is correlated to any other one, which implies that the compositions of the other eigenvectors are very different. For example, PC2 is highly influenced by Zero Bond and Zero Firm, whereas Turnover is almost the only variable that explains PC3.

Our results are slightly different from those obtained by Dick-Nielsen et al. (2012) and Dionne and Maalaoui Chun (2013). In the first case, the authors obtained a composite index of the Amihud, Amihud Risk, IRC and IRC Risk measures. In the second case, they found a composite index of the IRC, IRC Risk and Roll measures. The differences may be explained by the analysis of different periods and by the composition of bonds used. Notably, Dick-Nielsen et al. (2012) worked with a sample that covered January 1, 2005 to June 30, 2009, whereas the sample examined by Dionne and Maalaoui Chun (2013) ranges from July 1, 2002 to December 31, 2012. In both cases, the authors studied a combination of bonds with different credit ratings, whereas our investigation focuses on BBB bonds.

6.2 Analysis of the illiquidity index

Figure 1 below shows a graphical representation of the median monthly values of our illiquidity risk measure (Rliq). The graphical representation of the five illiquidity measures retained in PC1 are presented in Appendix A.3.

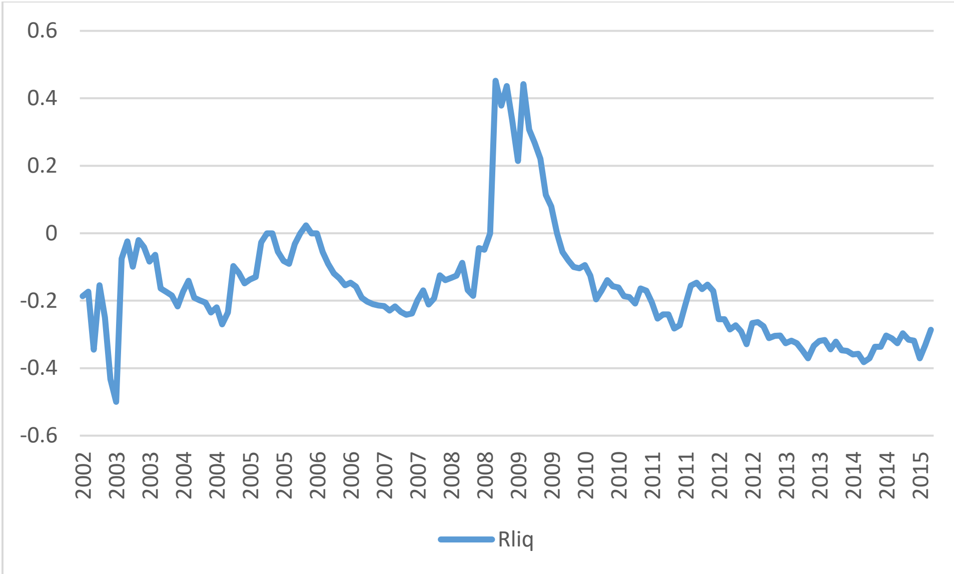

Figure 1: Time series of the Rliq measure between July 2002 and April 2015

We can see various spikes in the series. Notably, Figure 1 shows the spikes associated with the GM/Ford crisis that affected the US bond market in 2005-2006. This is reflected by the fact that Rliq approaches the value of zero, which is higher than the mean of around -0.2. The largest spikes observed are those that correspond to the recent financial crisis, which occurred in late 2008 and in 2009, with values that exceed 0.4. This clearly underlines the importance of liquidity risk on the US bond market during the last financial crisis. Following these spikes, our measure drops quickly in the second half of 2009, and then returns to precrisis levels followed by a gradual slight decline. Therefore, the measure drops below the -0.2 level in 2011 and continues this downward trend. These results clearly indicate that BBB bonds lose illiquidity after 2010. The TARP intervention started in October 2008, when illiquidity converged to its highest value. The TARP program thus seems to have restored liquidity in the market very efficiently, at least for BBB bonds.

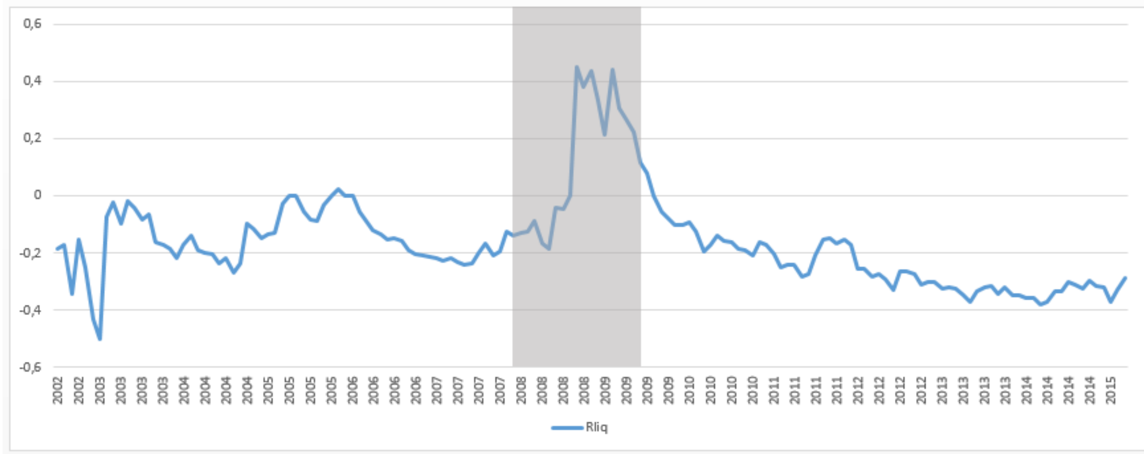

Figure 2: Rliq measure observed relative to the NBER recession
(December 2007-June 2009)

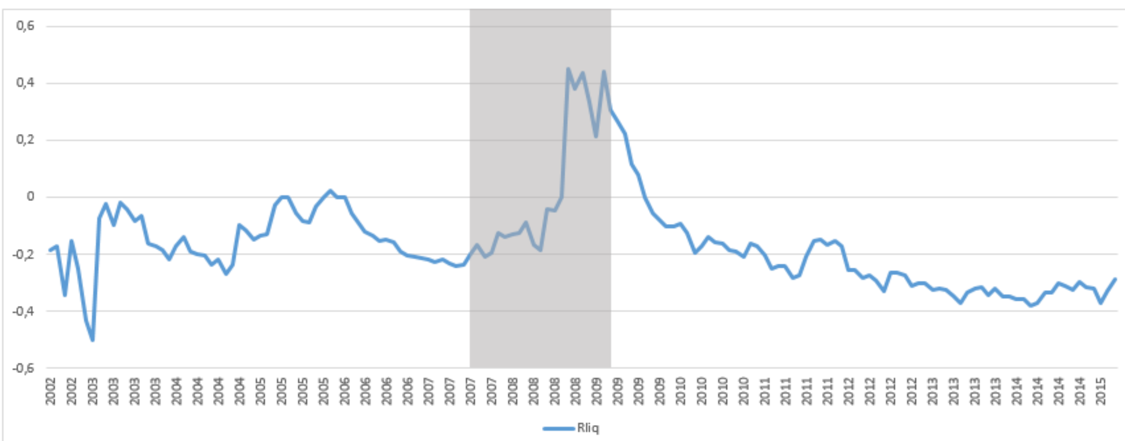

Figure 3: Rliq measure observed relative to the last financial crisis
(July 2007-March 2009)

In Figures 2 and 3, we observe the evolution of our index relative to the last NBER recession (December 2007-June 2009) and the last financial crisis (July 2007-March 2009). These results are interesting because first we can see that the crisis indeed began with an immediate increase in liquidity risk in the bond market. We observe the first spike in the beginning of 2008. This main spike seems to correspond to the Lehman Brothers bankruptcy in September 2008. The crisis ends with a last spike in bond liquidity risk, and the index then rapidly decreases to precrisis levels in early 2010. By comparison, the NBER recession began with a increase in bond liquidity risk and ended in the middle of a slump in bond liquidity risk. We can compare these results more extensively with the regime detection model.

6.3 Identification of illiquidity regimes

First, we apply the Markov model to the daily data of our measure Rliq, then to the monthly data to better represent the results graphically. We use the median of our daily data in order to obtain the monthly data.

Liquidity	Coefficient	Standard error
State1 μ_1	-0.2359	0.0031
State2 μ_1	0.0672	0.0068
Sigma	0.1416	0.0018
p11	0.9988	0.0007
p21	0.0047	0.0026
Sample: 2002-07-01 − 2015-03-31		
Number of states = 2		
Log likelihood = -1692.7461		

Note: State 1: $Rliq_t = \mu_1 + \varepsilon_1$; State 2: $Rliq_t = \mu_2 + \varepsilon_2$.

Table 5: Results of daily data with Markov-switching dynamic regression

Table 5 presents the results with the daily data. The table shows that μ_1= -0.2359. We can characterize Regime 1 as a low liquidity risk regime. In addition, μ_2 = 0.6722, which corresponds to the high liquidity risk regime. After evaluating the likelihood function, we can determine the transition probabilities. Table 5 shows that p11, which is the probability of being in the low liquidity risk regime and remaining there for the next period, is 0.9988. Therefore, the probability of being in a low liquidity risk regime and going to a high liquidity risk regime, which we call p12, is 0.0012. Further, p22, which is the probability of being in a high liquidity risk regime and staying there for the next period, is 0.9953. In addition, the probability p21, which is to go from regime 2 to regime 1, is 0.0047. It is not surprising to see that the probabilities of changing regimes are very low, which emphasizes that regime changes are rare events.

Liquidity	Coefficient	Standard error
State1 μ_1	-0.1974	0.0092
State2 μ_1	0.2945	0.0373
Sigma	0.1090	0.0063
p11	0.9934	0.0066
p21	0.0967	0.0872
Sample: 2002m7 − 2015m3		
Number of states = 2		
Log likelihood = -113.86842		

Table 6: Results of monthly data with Markov-switching dynamic regression

The results of our monthly data, shown in Table 6, are similar to those of daily data. We observe that μ_1= -0.1974 in this case, which is close to the value found with the daily data. However, μ_2= 0.2945, which is far from the value found with the daily data, is probably explained by the use of monthly medians. Regarding the transition probabilities, p11 = 0.9934, p12 = 0.0066, p21 =0.0967, and p22= 0.9033. Interestingly, even if the probability of remaining in a low liquidity risk regime is still as high as with daily data, the probability of remaining within a high liquidity risk regime is lower. Therefore, monthly data indicate that remaining in a high liquidity risk regime is less likely to persist than with daily data. However, the probability is over 90%, which is still high.

6.4 Graphical representation of regimes

We examine the graphical representation of these results to draw other conclusions on the links between illiquidity regimes and both the last NBER recession and the last financial crisis. Given the values obtained by applying the model, we can predict the current regime over time. Note that we used the monthly medians of our daily values in the charts below.

Figure 4 shows the dates where the model allows us to predict a regime change. First, we can see that after a period of a low liquidity risk regime, the model predicts a change to a high liquidity risk regime in late 2008 at about the date of the Lehman bankruptcy. We thus switch from a regime where the liquidity risk of bond premiums is around -0.2 to one where it is around 0.2. Further, this high illiquidity risk regime ends in September 2009, and then returns to a low liquidity risk regime until the end of the sample period. Also note that although our measure Rliq indicates an increase in liquidity risk linked to the GM/Ford crisis, it does not seem sufficiently strong to signal a regime change. This observation accentuates the fact that the financial crisis was characterized particularly by a rise in liquidity risk, relative to what was observed in recent years in the US corporate bond market. We can consequently put these regime changes into context compared with the last financial crisis, along with the NBER recession.

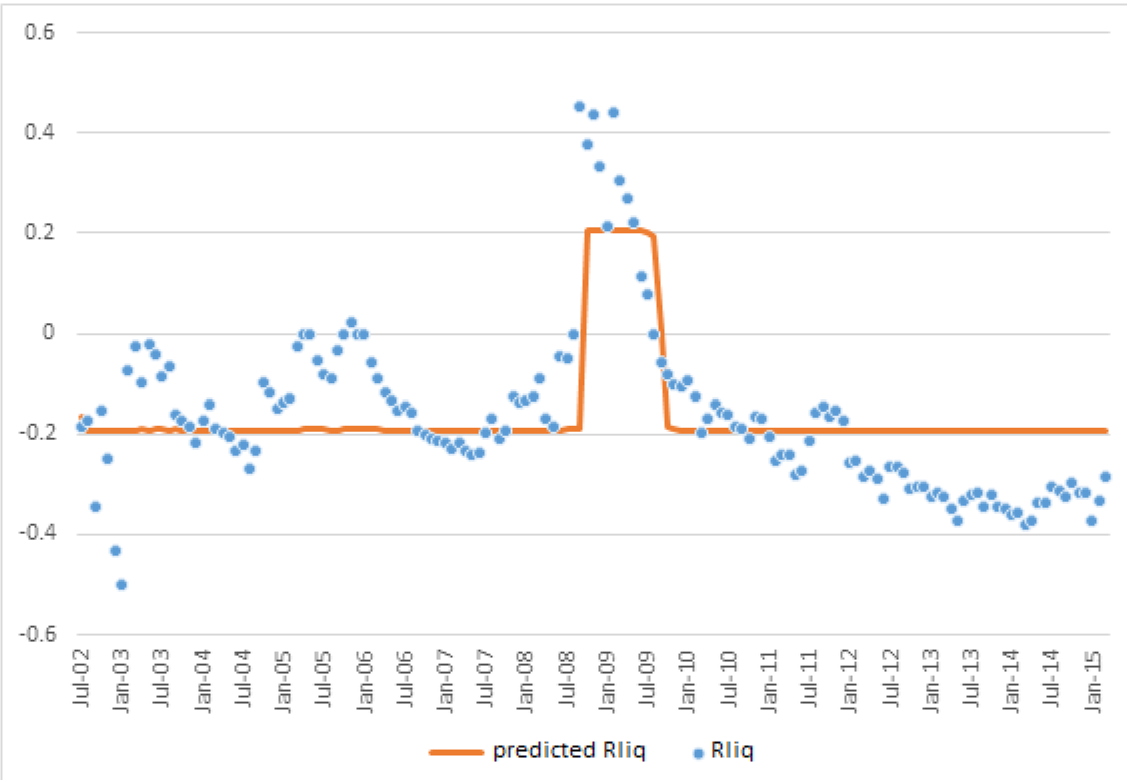

Figure 4: Regime detection using Rliq

Figure 5 shows that the bond regime changes more than a year after the crisis begins. Saunders and Allen (2010) and Dionne and Maalaoui Chun (2013) date the start of the crisis to July 2007. Dionne and Maalaoui Chun (2013) found that the last financial crisis began with a rise in liquidity risk, and hence observed the presence of a second regime starting in June 2008, which is not far from the date of September 2008.

These differences may be due to the Markov model, which is limited to two states or regimes, or to the fact that our sample comprises BBB bonds exclusively. The data in the study by Dionne et al. (2013) include B and BB bonds. In addition, the type of model detection regime in the study by Dionne and Maalaoui Chun (2013) differs from the Markov model. Those authors used a "regime shift" model, whereas our model is a Markov "regime switching" model.

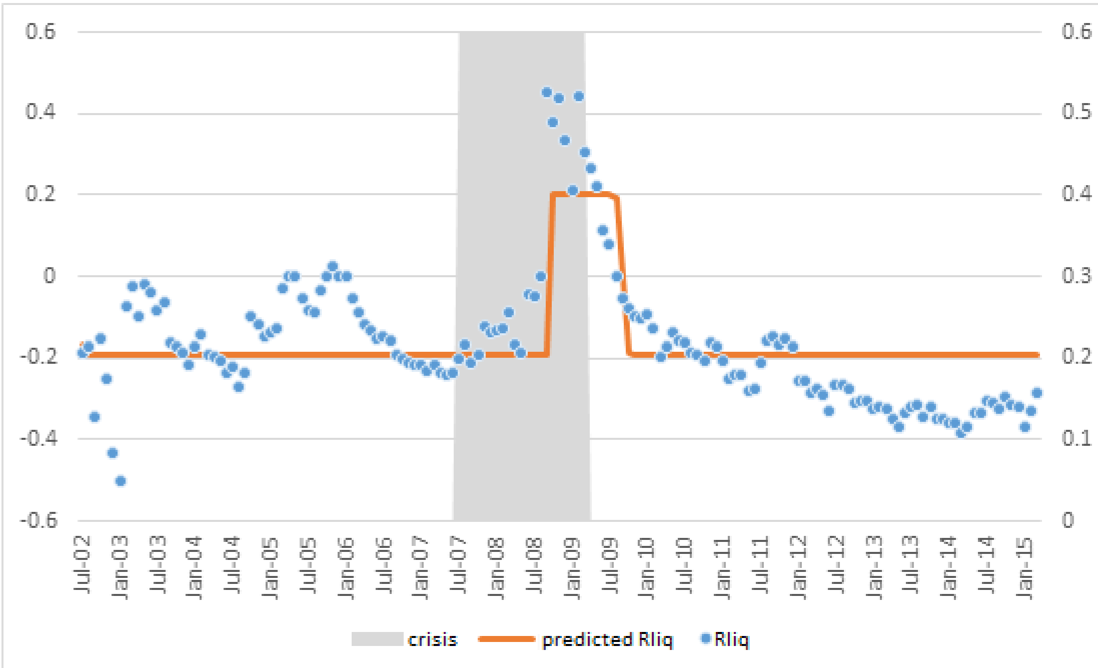

Figure 5: Illiquidity index during the financial crisis (July 2007-March 2009)

We now look at our regimes during the NBER recession. Notably, we can observe that the NBER recession began about one year before the appearance of our high liquidity risk regime for BBB bonds, which is a significant lag. The predictive nature of liquidity risk on the economic recession is refuted with the Markov model. However the NBER recession ended in June 2009, and our high liquidity risk regime declined a few months later, which may reveal the persistence of illiquidity after the recession.

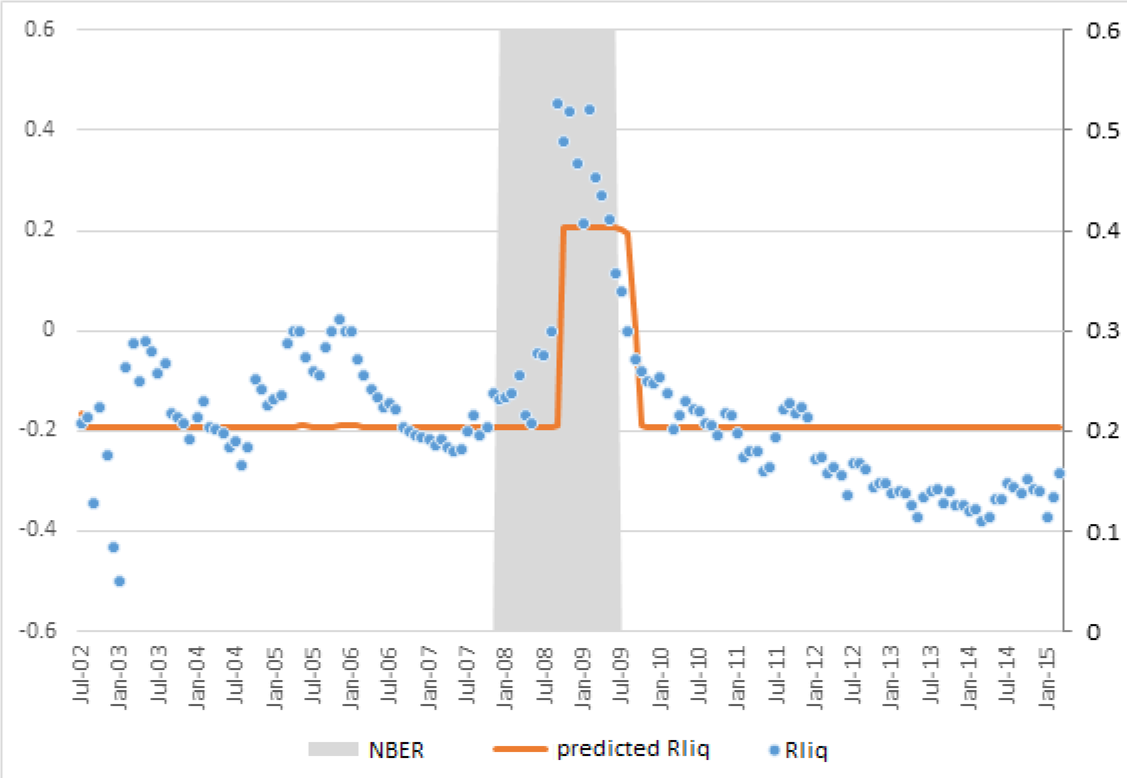

Figure 6: Illiquidity index during the NBER recession (December 2007-June 2009)

In a recent study, Bao et al. (2016) analyzed the effect of the Volcker Rule on bond liquidity. The objective of the Rule included in the Dodd-Frank Act was to limit banks' speculative activities. Using a difference-indifference analysis, Bao et al. (2016) studied whether illiquidity has become worse in periods of bond stress since the Volcker Rule (April 1, 2014). They also considered the new liquidity rules implemented by Basel III. They confirm that the introduction of the Volcker Rule reduced liquidity for BBB bonds exposed to stress events (downgraded from investment-grade BBB to speculative-grade BB), after the implementation of the Rule in April 2014. As Figure 6 indicates, we do not observe such an effect for the BBB bonds in our data probably because we did not keep downgraded BBB bonds in our sample. It seems that the Volker Rule did not affect the liquidity of non-stressed bonds significantly.

7. Conclusion

We applied the methodology framework used by Dick-Nielsen et al. (2012) to measure the liquidity risk of corporate bonds. We obtained a compound index of the Amihud, Amihud Risk, Roll, IRC and IRC Risk measures. This result differs slightly from those obtained by Dick-Nielsen et al. (2012) and Dionne and Maalaoui Chun (2013). In the first case, the authors found an index made up of the Amihud, Amihud Risk, IRC and IRC Risk measures. The second set of authors found an index composed of Roll, IRC and IRC Risk.

We have identified two liquidity risk regimes in our sample by applying the Markov model (1994). The advantage of using the Markov chain model to detect regimes is mainly its flexibility and simplicity to model a time series made up of discrete variables. We find that our first regime, namely the low liquidity risk regime, extends initially from the start of our sample period in July 2002 until September 2008. It then reappears from September 2009 until the end of our sample period in March 2015. The second regime therefore lasts from September 2008 to August 2009 and begins with the Lehman bankruptcy during the financial crisis and persists after the end of both the financial crisis and the NBER recession. These results do not affirm the predictive nature of liquidity risk regarding the NBER recession of 2007-2009, meaning that liquidity regimes must be distinguished from both economic regimes and default regimes. Therefore, capital management for liquidity risk must be based on liquidity risk per se. Moreover, regime detection techniques must be included in banks' optimal capital management for both liquidity risk and credit risk.

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Appendix

A.1 Markov chains

Let s_t be a random variable that can take values in {1,2, …, N} only. We assume that the probability that the unobservable state variable s_t is equal to *j* depends solely on the most recent value s_{t-1}. Therefore:

$$P\{s_t = j \mid s_{t-1} = i, s_{t-2} = k, \ldots\} = P\{s_t = j \mid s_{t-1} = i\} = p_{ij.} \tag{A1}$$

p_{ij} is defined by the probability that the state *i* is followed by the state *j*. p_{ij} is thus a transition probability from a Markov chain with N states and is between 0 and 1. One of the properties of transition probabilities is that:

$$p_{i1} + p_{i2} + \ldots + p_{iN} = 1.$$

In addition, these transition probabilities can be represented in a transition matrix (N x N):

$$\mathbf{P} = \begin{bmatrix} p_{i1} & p_{21} & \cdots & p_{N1} \\ p_{12} & p_{22} & \cdots & p_{N2} \\ \vdots & \vdots & \cdots & \vdots \\ p_{iN} & p_{2N} & \cdots & p_{NN} \end{bmatrix}. \tag{A2}$$

Another way to represent a Markov chain is to use the random vector (N x 1) for which the *j*th element is equal to 1 if $s_t = j$ and is equal to 0 otherwise. Consequently, we can represent the vector as follows:

$$\boldsymbol{\xi}_t = \begin{cases} (1, 0. 0. \ldots, 0)' \ when \ s_t = 1 \\ (0. 1, 0. \ldots, 0)' \ when \ s_t = 2 \\ \vdots \\ (0. 0. 0. \ldots, 1)' \ when \ s_t = N \end{cases} \tag{A3}$$

We can then see that when s_t = 1, the vector $\boldsymbol{\xi}_t$ is equal to the first column of the identity matrix $\boldsymbol{I}_N$ (N x N) and so on.

If $s_t = i$, then the *j*th element of $\boldsymbol{\xi}_{t+1}$ is a random variable that takes the value of 1 with the probability p_{ij} and is equal to 0 otherwise. The mean of this random variable is therefore p_{ij}. Its conditional mean at $s_t = i$ is given by:

$$\text{E}\ (\boldsymbol{\xi}_{t+1} |\ s_t = \text{i}) = \begin{bmatrix} p_{i1} \\ p_{i2} \\ \vdots \\ p_{iN} \end{bmatrix}, \tag{A4}$$

which is also the *i*th column of the **P matrix.** Further, given that the vector $\boldsymbol{\xi_t}$ corresponds to the *i*th column of $\boldsymbol{I}_N$ when $s_t = i$, then (A4) can be written as:

$$\text{E}(\boldsymbol{\xi}_{t+1} |\ \boldsymbol{\xi}_t) = \mathbf{P}\ \boldsymbol{\xi}_t.$$

In addition, owing to the Markov property we obtain:

$$\text{E}(\boldsymbol{\xi}_{t+1} |\ \boldsymbol{\xi}_t, \boldsymbol{\xi}_{t-1}, \ldots) = \mathbf{P}\ \boldsymbol{\xi}_t.$$

The Markov chain can therefore be expressed by:

$$\boldsymbol{\xi}_{t+1} = \mathbf{P}\ \boldsymbol{\xi}_t + \boldsymbol{v}_{t+1} \tag{A5}$$

where $\boldsymbol{v}_{t+1} = \boldsymbol{\xi}_{t+1} - \text{E}\ (\boldsymbol{\xi}_{t+1} |\ \boldsymbol{\xi}_t, \boldsymbol{\xi}_{t-1}, \ldots).$

From expression (A5) we obtain:

$$\boldsymbol{\xi}_{t+m} = \boldsymbol{v}_{t+m} + \mathbf{P}\ \boldsymbol{v}_{t+m-1} + \boldsymbol{P}^2\boldsymbol{v}_{t+m-2} + \ldots + \boldsymbol{P}^{m-1}\boldsymbol{v}_{t+1} + \boldsymbol{P}^m\boldsymbol{\xi}_t \tag{A6}$$

where $\boldsymbol{P}^m$indicates the transaction matrix multiplied by itself m times. Therefore, the prediction of a Markov chain in m periods can be obtained with:

$$\text{E}\ (\boldsymbol{\xi}_{t+m} | \boldsymbol{\xi}_t, \boldsymbol{\xi}_{t-1}, \ldots) = \boldsymbol{P}^m\boldsymbol{\xi}_t. \tag{A7}$$

Specifically, the probability that an observation of regime *i* is followed m periods later by an observation of regime *j*, P (s_{t+m}= j | s_t = i), is given by line *j* and column *i* of the $\boldsymbol{P}^m$ matrix.

An irreducible Markov chain with N states and a transition matrix **P** is called ergodic if one of the eigenvalues of **P** is 1 and all the other eigenvalues of **P** fall within the unit circle. The vector of (N x 1) of the ergodic probabilities of an ergodic chain is noted as $\boldsymbol{\pi}$ and satisfies:

$$\mathbf{P}\boldsymbol{\pi} = \boldsymbol{\pi} \tag{A8}$$

The vector $\boldsymbol{\pi}$ is defined as the eigenvector of $\mathbf{P}$ associated with a unitary eigenvalue. This eigenvector is unique and can be seen as indicating the unconditional probability of each different state N. It is also called the unconditional probability vector.

A.2 Structure of Hamilton's model (1994)

We use a dynamic regression model representation. In the case of a dynamic regression, the model adjusts more quickly when the process changes regimes. It is also more suited to higher frequency data than an autoregressive model is. The general model using a dynamic regression can be represented as:

$$y_t = \mu_s + \boldsymbol{x}_t\boldsymbol{\gamma} + \boldsymbol{z}_t\boldsymbol{\beta}_s + \varepsilon_s \tag{A9}$$

where y_t is the dependent variable, μ_s is the dependent intersection of the regime, $\boldsymbol{x}_t$ is a vector of exogenous variables with parameters ($\boldsymbol{\gamma}$) that do not depend on s, $\boldsymbol{z}_t$ is another vector of exogenous variables with parameters ($\boldsymbol{\beta}_s$) that are a function of s, and ε_s is the normal error term i.i.d., with a mean of 0 and dependent variance of the regime is σ_s^2. To estimate the model, we can use the Maximum Likelihood Method.

We can write the conditional density function of y_t as:

$$\mathrm{f}\,(\boldsymbol{y}_t|s_t = j, \boldsymbol{x}_t, \boldsymbol{Y}_{t-1}; \boldsymbol{\alpha}) \text{ for all } j \tag{A10}$$

where $\boldsymbol{\alpha}$ is a vector of the parameters that characterize the conditional density function, $\boldsymbol{y}_t$ is a vector of the endogenous variables, and $\boldsymbol{x}_t$ is a vector of the exogenous variables. $\boldsymbol{Y}_{t-1}$ is a vector containing all past observations of $\boldsymbol{y}$ and $\boldsymbol{x}$. Therefore, the marginal density function of $y_t$is obtained by weighting the different conditional density functions by their respective probabilities. We obtain:

$$\mathrm{f}\,(y_t|\,\boldsymbol{\alpha}) = \sum_{j=1}^{N} \mathrm{f}(y_t|s_t = j, x_t, Y_{t-1}; \boldsymbol{\alpha})\ \Pr(s_t = j; \boldsymbol{\alpha}). \tag{A11}$$

The likelihood function requires us to estimate the probability that s_t takes a specific value by observing the data until time *t;* we must also model the $\boldsymbol{\alpha}$ parameters.

Let $\Pr(s_t = j|y_t; \boldsymbol{\alpha})$ be the conditional probability of identifying $s_t = j$ by observing the data until time *t*. Therefore:

$$\Pr(s_t = j|y_t; \boldsymbol{\alpha}) = \frac{\mathrm{f}(y_t|s_t = j, x_t, Y_{t-1}; \boldsymbol{\alpha})\Pr(s_t = j|Y_{t-1}; \boldsymbol{\alpha})}{\mathrm{f}(y_t|x_t, Y_{t-1}; \boldsymbol{\alpha})} \quad \text{(A12)}$$

where $\mathrm{f}\,(y_t|x_t, Y_{t-1}; \boldsymbol{\alpha})$ is the likelihood function of y_t and $\Pr(s_t = j|Y_{t-1};\ \boldsymbol{\alpha})$ is the anticipated probability of $s_t = j$ given the observations until time t-1. Consequently:

$$\Pr(s_t = j|y_{t-1}; \boldsymbol{\alpha}) = \sum_{j=1}^{N} \Pr(s_t = i|s_{t-1} = j; y_{t-1}; \boldsymbol{\alpha})\Pr(s_{t-1} = j|y_{t-1};\ \boldsymbol{\alpha}). \quad \text{(A13)}$$

We note as $\boldsymbol{\xi}_{t|t}$ and $\boldsymbol{\xi}_{t|t-1}$the vectors (k x 1) of conditional probabilities $\Pr(s_t = j|y_t; \boldsymbol{\alpha})$ and $\Pr(s_t = j|y_{t-1}; \boldsymbol{\alpha})$ respectively. The likelihood function is obtained by iteration.

The maximum likelihood function can be written as:

$$\mathrm{L}(\boldsymbol{\alpha}) = \sum_{t=1}^{T} \log f(\boldsymbol{y}_t \mid \boldsymbol{x}_t, \boldsymbol{Y}_{t-1}\,;\ \boldsymbol{\alpha}) \quad \text{(A14)}$$

where $$\mathrm{f}\,(\boldsymbol{y}_t \mid \boldsymbol{x}_t, \boldsymbol{Y}_{t-1}\,; \boldsymbol{\alpha}) = \mathbf{1}'\,(\boldsymbol{\xi}_{t|t-1} \odot \boldsymbol{\eta}_t)$$

and $\boldsymbol{\eta}_t$ is the vector of conditional density functions. In our application, we do not use control variables and we assume there are only two states, so $y_t = \mu_s + \varepsilon_s,\ s = 1{,}2$.

A.3 Retained components of the illiquidity index

Amihud

To obtain figures A.3.1 to A.3.5 in this section, we calculated the monthly median values of each measure.

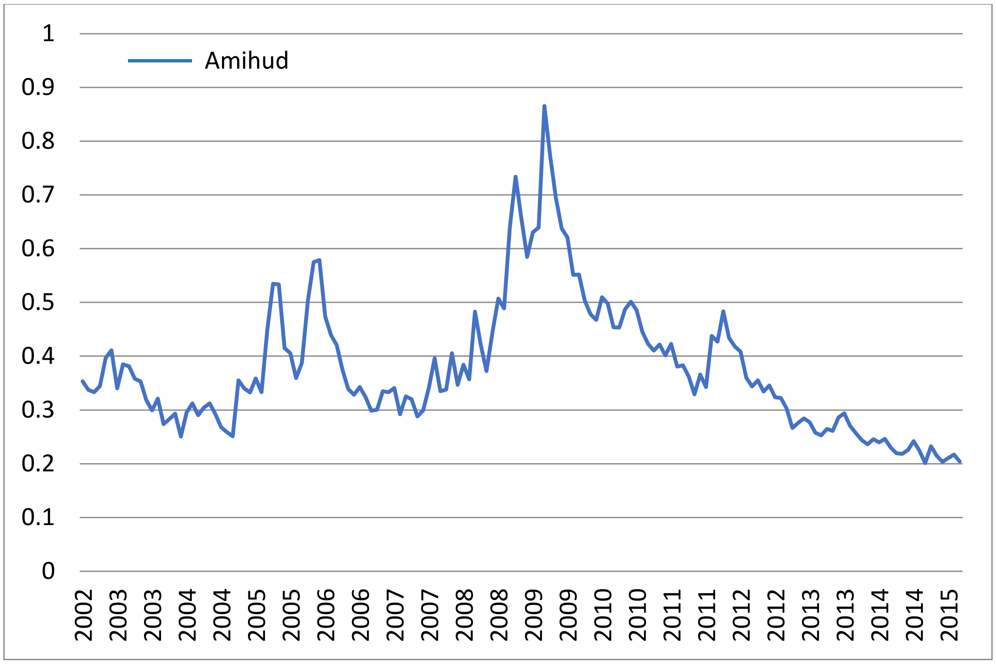

Figure A.3.1: Time series of the Amihud measure between July 2002 and April 2015

Figure A.3.1, which represents the Amihud measure in our sample, illustrates spikes in liquidity risk. The two highest spikes occurred in late 2008-2009 with a respective value of about 0.72 and 0.88, which would coincide with the last financial crisis. These results are therefore in line with previously reported observations by Dick-Nielsen et al. (2012) and Maalaoui Chun and Dionne (2013), which demonstrate an increase in liquidity risk during the financial crisis. In addition, we can see two other peaks of lesser amplitude between 2005 and 2006, with values of about 0.53 and 0.58. This period corresponds to the crisis that affected General Motors (GM) and Ford in 2005: their bonds were downgraded to junk status. The results of the Amihud measure suggest that this crisis indeed caused an increase in the liquidity risk on the US corporate bond market. Lastly, we can see that in late 2013, the Amihud measure fell to a lower level than previously observed, and decreases until the end of the sample period.

<u>Amihud Risk</u>

The Amihud Risk measure represents the standard deviation of the Amihud measure. As Figure A.3.2 indicates, two spikes occur during the financial crisis, with values approaching 1.2 and 1. The first peak is higher than the second, contrary to the pattern of the Amihud measure. Spikes also occur in 2005-2006, with values approaching 0.7 and

0.8, which corresponds to the GM/Ford crisis. Here again, we can see that after the financial crisis, the Amihud Risk gradually decreases until the end of the sample period.

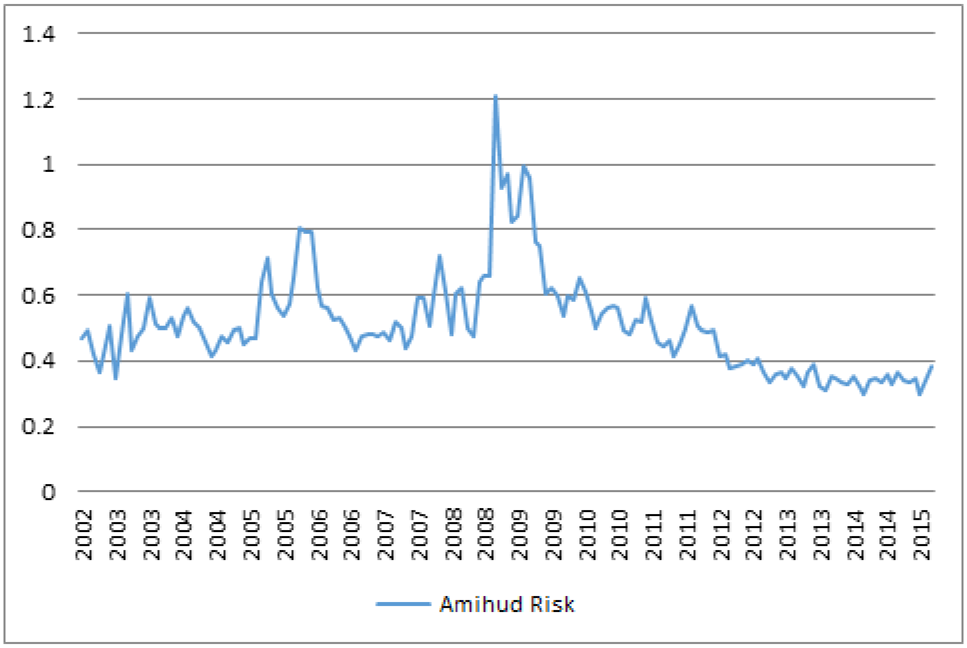

Figure A.3.2: Time series of the Amihud Risk measure between July 2002 and April 2015

IRC

Figure A.3.3 illustrates the results produced by the IRC measure. The measure is slightly different in form from those of the previous charts. There are still spikes during the last financial crisis, but they seem to be represented by a single, longer lasting spike. The spikes of the GM/Ford crisis of 2005-2006 also seem to last longer. However, starting in late 2011, we can also note a decline in the value of the IRC measure, which continues until the end of the sample period. It is therefore interesting to compare these results with those of the standard deviation of IRC.

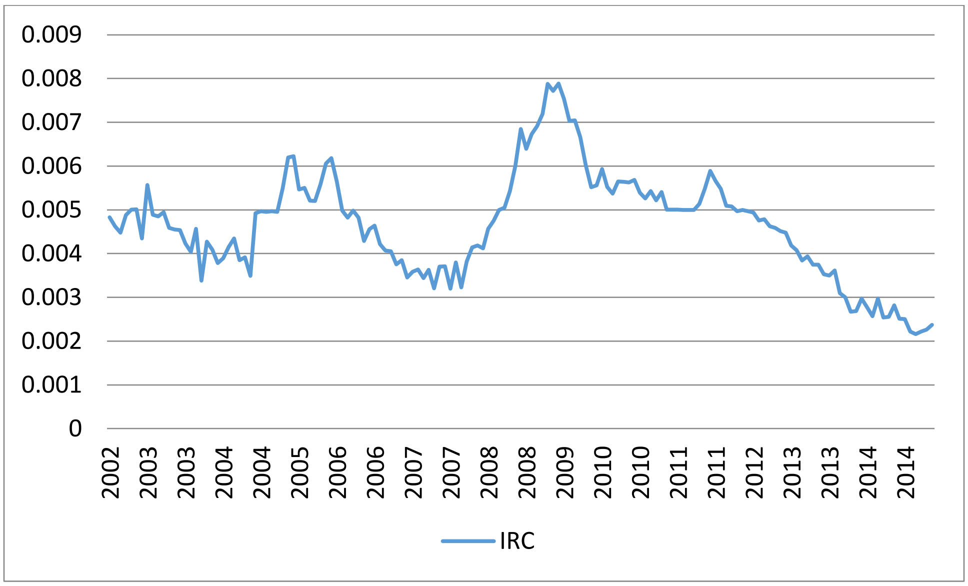

Figure A.3.3: Time series of the IRC measure between July 2002 and April 2015

IRC Risk

Figure A.3.4 presents the results of the calculations of the IRC Risk measure. It shows the spikes of the last financial crisis, with values close to 0.008 and 0.007. Here again, we observed the spikes of the GM/Ford crisis of 2005-2006, with values approaching 0.006 and 0.007. However, the difference between the spikes of these two crises seems lesser than in the other risk measures. In addition, this risk stabilizes after the financial crisis instead of decreasing, as it does for the previous measures.

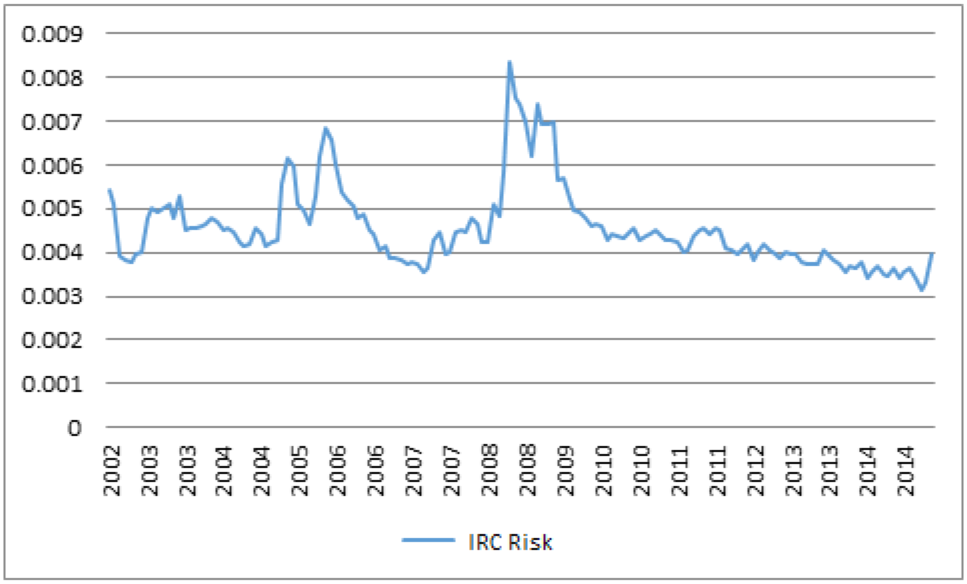

Figure A.3.4: Time series of the IRC Risk measure between July 2002 and April 2015

<u>Roll</u>

The Roll measure produces results, shown in Figure A.3.5, that are consistent with those observed for the Amihud and IRC measures. We can see the spikes of 2008-2009 with values between 2.4 and 2.6, and those of the GM/Ford crisis in 2005-2006, whose amplitude seems similar to that of the previous spikes, at around 1.6-1.7. However, the spike in 2003 reaches a value of nearly 1.8, and is therefore higher than those of the GM/Ford crisis, for the first time.

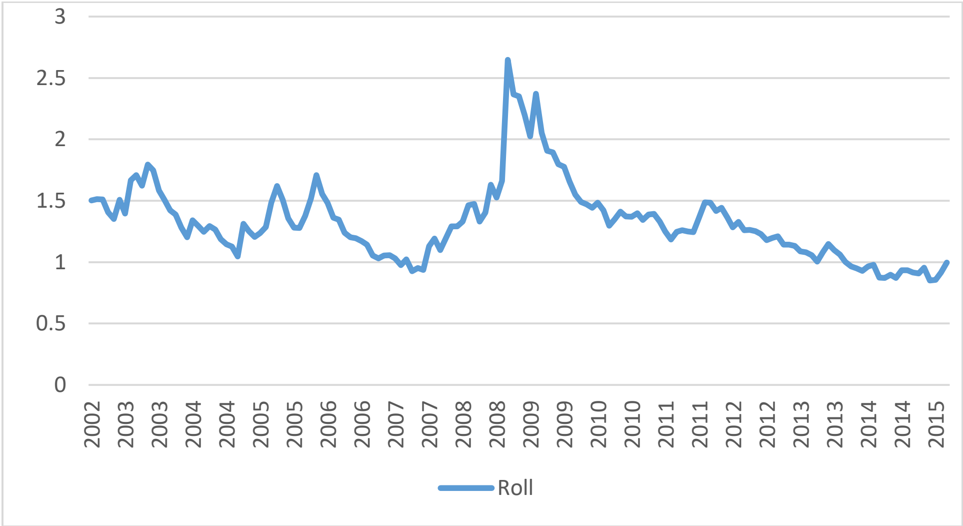

Figure A.3.5: Time series of the Roll measure between July 2002 and April 2015