# Proper Risk Behavior

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**Abstract**

How does risk aversion affect choices when expenses improve probabilities? Attempts to answer this question in the literature found an endogenous switching probability. In this paper we introduce a new concept of comparative attitude to risk, namely proper risk behavior and determine ½ as the threshold probability over which a more proper risk behavior agent becomes a gambler. We consider applications related to self-protection and willingness to pay. We give a sufficient condition for analytic comparative proper risk behavior and show that all results hold in the presence of a background risk.

*Keywords*: Mixed risk aversion, proper risk aversion, proper risk behavior, self-protection, willingness to pay, background risk, principal-agent.

**Résumé**

Comment la riscophobie peut-elle affecter les choix lorsque les dépenses affectent les probabilités ? Des essais pour répondre à cette question ont proposé une probabilité endogène cible. Dans cette étude, nous introduisons un nouveau concept, soit le comportement cohérent face au risque, et déterminons ½ comme étant la probabilité cible au-delà de laquelle un agent qui a un comportement plus cohérent devient un joueur. Nous considérons également des applications reliées à la prévention et à la volonté à payer. Nous proposons une condition suffisante pour comparer les divers degrés de comportement cohérents et nous montrons comment nos résultats peuvent être étendus à des situations avec deux sources de risques.

*Mots clés* : Aversion au risque mélangée, aversion au risque cohérente, comportement cohérent face au risque, autoprotection, volonté à payer, principal agent.

# 1 Introduction

For many economic applications under risk and uncertainty, a simple concave transformation of a von Newmann-Morgenstern utility function (or an Arrow-Pratt increase in risk aversion) does not yield intuitive changes in decision variables or in lottery choices by risk averse individuals. For example, Ross [1981] showed that the risk premium of a more risk averse agent may not be larger than that of a less risk averse agent in the presence of a background risk or that a more risk averse individual may choose a more risky portfolio in the same environment.

In another example, following the contribution of Ehrlich and Becker [1972] who introduced the concepts of self-protection and self-insurance in the literature, Dionne and Eeckhoudt [1985] showed that a more risk averse individual does not necessarily produce more self-protection activities than a less risk averse one[1]. In fact, one cannot make any prediction on how a more risk averse agent will choose his optimal level of effort in a principal-agent relationship without introducing strong assumptions such as the separability of the utility function (Arnott, 1992).

A third example concerns the willingness to pay literature (Drèze, 1962; Jones-Lee, 1974; and Pratt and Zeckhauser, 1996). One can easily verify that a more risk averse decision maker in the sense of Arrow-Pratt is not necessarily willing to pay more for a lower probability of death or for a lower probability of accident than a less risk averse decision maker (Eeckhoudt, Godfroid and Gollier, 1997). In a fourth example, McGuire, Pratt and Zeckhauser [1991] showed that more risk averse individuals may choose more risky decisions (described as less insurance and more gamble) than less risk averse individuals. They veri ed that these behaviors are function of a critical switching probability.

In the three examples discussed in the two preceding paragraphs, the individuals decisions imply rst order shifts instead of pure second order ones. Moreover, as we will see, their actions usually affect higher moments when appropriate restrictions are not imposed[2]. Consequently, to make predictions

[1] On this issue see also Briys and Schlesinger [1990], Julien, Salanié and Salanié [1998] and Chiu [1997].

[2] For the self-protection example, the $i^{th}$ moment of the gross expected loss is $p(x)\,l^i$, where $p$ is the probability of accident, $x$ is the level of self-protection and $l$ is the amount of loss in case of accident.

on (risk averse) decision makers behaviors, one needs restrictions either on utility functions or on distribution functions that take into account all distribution moments that are modified by the individuals choices. In this paper, we shall concentrate on restrictions related to utility functions. For an analysis of restrictions on distribution functions see Julien, Salanié and Salanié [1998], and for restrictions on the loss function see Lee [1998].

In 1987, Pratt and Zeckhauser introduced the concept of Proper Risk Aversion in order to make prediction of lottery choices in presence of an independent, undesirable lottery or of an independent background risk. Their concept is preserved in the class of utility functions that are completely monotone or whose derivatives alternate in sign, with positive odd derivatives and negative even derivatives. These functions come from a mixture of risk averse exponential utility functions. Brocket and Golden [1987] developed a parallel characterization of such functions and Hammond [1974] proposed a first application using a mixture (discrete) of exponential functions.

Recently, Caballé and Pomansky [1996] extended the analysis by characterizing stochastic dominance in presence of such functions. They applied their model to the standard portfolio choice and provided a new set of sufficient conditions to obtain that a mixed risk averse individual will decrease his risky position when the risk increases. One can also show that simple concave transformations of mixed risk aversion functions are sufficient to make comparison of different risk averse individual choices for this simple portfolio problem without a background risk and where the decision variable does not affect the mean of the random variable.

However, up to now, no study has proposed a transformation of the utility function that would permit comparison of individual decisions that affect all the moments of the distribution. The objective of this paper is to propose such a transformation for mixtures of exponential utilities.

In Section 2, we discuss on how the concept of mixed risk aversion is useful to compare the levels of self-protection between a risk averse agent and a risk neutral one. We first obtain that there exists an endogenous probability such that a risk averse individual will produce more self-protection activities than a risk neutral one[3]. A more interesting result is to find an exogenous bound for such probability. In fact, under mixed risk aversion, this threshold probability will be shown to be lower than 1/2. We also obtain that the threshold

[3]Jullien, Salanié and Salanié [1998] derived simultaneously and independently an identical result.

probability is equal to 1/2 when the utility function is quadratic. Two direct extensions of these results will imply that the switching probability de ned in Mcguire, Pratt and Zeckhauser [1991] is greater than 1/2 under mixed risk aversion[4] and that the willingness to pay threshold is also lower than 1/2 when we compare the choice of a risk averse individual to that of a risk neutral one.

However, mixed risk aversion is not sufficient to compare such decision variables between different risk averse individuals. In Section 3 we propose the concept of Proper Risk Behavior. We use the term Proper Risk Behavior since our concern is to compare optimal decision variables that affect all the moments of the random variable distribution. We apply this new concept to the class of mixed risk averse functions.

By de nition, individual $v$ has a more proper risk behavior than individual $u$ if he is more risk averse, more prudent, more temperent ... or if the absolute ratio of the $n^{th+1}$ derivative of $v$ over the $n^{th}$ is higher than the corresponding ratio of individual $u$ for all $n$ greater than one. We provide different characteristics of the proper risk behavior function and we obtain that many utility functions share the notion of proper risk behavior.

Among other results, we will show that the threshold probability where individuals having a more proper risk behavior will produce more self-protection activities or will be willing to pay more for lower probabilities of accidents remains lower than 1/2. This result is important since the great majority of risky situations that include self-protection and public decisions on safety are characterized for events with probability lower than 1/2. We also obtain that the switching probability to become a gambler remains greater than 1/2 in the probability-improving environment of McGuire, Pratt and Zeckhauser. Finally, we extend the concept of Proper Risk Behavior to risky situations with a background risk (Doherty and Schlesinger, 1983).

[4]In their model, activity $x$ increases the winning probability instead of decreasing the probability of loss as in the self-protection and willingness to pay applications.