Dynamic Financial Contract under Extended Liability

by Bénédicte Coestier

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Bénédicte Coestier is assistant professor of economics, Université de Paris X-Nanterre and THEMA.

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Abstract

We consider an entrepreneur looking for external financing who may face two types of independent financial risks: a risk associated with its activity and a risk associated with an environmental accident. Using a costly state verification model, we characterize the optimal two-period financial contract under a law on liability allowing for extended liability to lenders. The optimal financial contract may be interpreted as a bond contract incorporating, among others, a compulsory insurance covenant. And under extended liability, the total cost of damage is internalized by both the entrepreneur and the investor.

Keywords: contract form, environmental risk, extended liability, compulsory insurance covenant, transaction costs.

JEL Classification: D23, D82, G22, G33, K32

Résumé

Nous considérons un entrepreneur à la recherche d'un financement externe pour un projet risqué. Cet entrepreneur fait face à deux types de risques financiers indépendants : un risque associé à son activité et un risque associé à un accident environnemental. Dans le cadre d'un modèle CSV, nous caractérisons le contrat financier de deux périodes optimal sous l'hypothèse selon laquelle l'entrepreneur est soumis à une règle de responsabilité autorisant l'extension de la responsabilité aux prêteurs. Le contrat financier optimal s'apparente à un contrat d'obligation comprenant, entre autres, une clause d'assurance obligatoire. L'extension de la responsabilité garantit que le coût total du dommage est internalisé en totalité par l'entrepreneur et l'investisseur.

Mots clés : forme de contrat, risque environnemental, responsabilité étendue, assurance obligatoire, coûts de transaction.

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1 Introduction

In a context of harmonization of environmental legislation at the European level, the European Union is seeking to adopt a law on liability for environmental damage, either by joining the Council of Europe Convention on Civil Liability for Damage Resulting from Activities Dangerous to the Environment or by way of a European directive. The choice of the litigation approach to manage environmental risks has been made for at least twenty years by the United States, the Comprehensive Environmental Response, Compensation and Liability Act of 1980 (CERCLA) and the toxic tort system being the most important systems of environmental liability in the US.[1] Liability for damages generally has two goals: to reduce risks by creating incentives to reduce the probability or magnitude of damages and to provide compensation to victims. But as the American experience demonstrates, the litigation approach to manage environmental risks appears to involve high transaction costs for few results with respect to risk reduction and victims compensation.[2]

CERCLA, the US federal legislation for the clean-up of dangerous hazardous waste sites better known as Superfund, creates a regime of strict,

[1] Cf. P. S Menell (1991) for a description of these two systems of environmental liability.

[2] D. R. Anderson (1998) proposes an assessment of environmental liability risk management and insurance in the US.

joint, several and retroactive liability. This particular liability rule greatly increases risk exposure of insurers and investors. In some recent judgments, investors who financed firms causing environmental damages have been found liable under CERCLA for part of the recovering costs.[3] This suggests that from now on investors may be exposed to a financial risk associated with an environmental accident and that they seek techniques to prevent supporting such a risk. Indeed, lending institutions have started to react to this financial risk by asking more information on environmental matters of a project and by strengthening loan conditions.

The purpose of this article is to analyze the impact of a financial risk associated with environmental damages on the design of financial contract, assuming the existence of a liability rule allowing for extended liability to lenders. The economic analysis of extended liability as such proceed from the "judgment proof" problem analyzed by Shavell (1986). A party is said to be judgment proof when he is unable to pay fully the amount for which he is liable. Extending liability then appears naturally as a solution to cost internalization and victims compensation. The objective of extended liability is to force the responsible party to internalize the total cost of risks and to reduce risks. When it is extended to lenders, lending institutions are chosen to implement this environmental risks management.[4] One can expect extended liability to have some impacts on the incentives to take care, on the cost of capital, on the availability of credit, and on the structure of financial contracts.

In the context of a principal-agent relationship, Pitchford (1995) shows that increasing lenders' liability can result in a higher accident probability and reduced social welfare. In addition, he analyzes the extent to which lenders should be held liable and he establishes that efficiency can be enhanced by choosing some partial lender liability (partial compensation by the lender). Using a multi-principal model involving a bank and an insurance sector, Boyer and Laffont (1997) study the interaction between banking contracts which determine financial constraints and the behavior of firms in their choice of insurance and safety activities. When the bank is informed

[3] For a description of various court cases, see Boyer and Laffont (1996).

[4] Liability can also be extended to buyers in case of property transfers (See Segerson (1992) and (1993)). Segerson (1992) more particularly analyzes the impact of alternative assignments of liability on environmental assessment and efficiency of property transfers. In a vertical relationship, "contractors" with the injurer may also be held liable (See Segerson (1990), Boyd and Ingberman (1996) and (1997)).

of both the levels of profits and the level of effort of the firm but the insurer is not informed, they show that unless insurance is compulsory, because of risk-neutrality and limited liability, the firm will not want to become insured and furthermore may choose to exert insufficient effort to reduce the probability of an environmental accident. The social cost of an accident being not fully internalized, the bank will overinvest. However, first-best levels of effort and lending may be achieved either by making the bank fully liable for damages or by making full insurance compulsory for the bank rather than for the firm. Under adverse selection or moral hazard for the lending institution, some informational rents must be given up to the firm in order to induce either truthful revelation of profits levels or effort. Under these asymmetric information contexts, the bank has a tendency to underinvest or to induce insufficient safety care, and liability of banks is not a sufficient instrument to reach the second-best allocation that would be chosen by a regulator. Boyer and Laffont also favor a partial liability rule for lenders. Considering the market for loanable funds, Heyes (1996) shows that extending liability to lenders, because of simultaneous problem of moral hazard and adverse selection, has a qualitatively ambiguous impact on the equilibrium cost of capital. And the conventional wisdom according to which extension of liability to lenders leads to an increase of interest rates is effective if adverse selection considerations dominate moral hazard ones. With respect to the availability of credit, it is highlighted that an increase in credit-rationing can co-exist with a decrease in the interest rate charged to borrowers.

In this paper, we choose to focus our attention on the impact of extended liability on the structure of financial contracts. More precisely, we seek to evaluate to what extent extended liability can affect the contractual relationship between an entrepreneur with a risky investment project looking for external financing and a lending institution. During the realization of the project, the entrepreneur may face two types of independent financial risks: a financial risk associated with the activity and a financial risk associated with an environmental accident (e.g., clean-up costs of pollution damages). Our objective is to highlight the consequences of incorporating this latter financial risk under a particular liability rule (allowing for extended liability to lenders) on the design of financial contract. Using a dynamic costly state verification model based on Chang (1991) and Caillaud, Dionne and Jullien (2000), we establish that a financial risk associated with environmental damages is covered by the investor selling insurance to the entrepreneur. More precisely, the optimal financial contract takes the form of a bond contract contingent on the occurrence of an environmental accident: insurance is then provided directly by the lending institution. This demand of insurance is motivated by the existence of transaction costs (costly auditing). When financing and insurance activities are supposed to be provided by two separate institutions both evolving in a competitive environment, this contingent bond contract can be interpreted as a non-contingent bond contract incorporating a covenant of compulsory insurance. Moreover, this financial risk coupled with a liability rule allowing for extended liability to lenders increases the cost of debt capital. A regulation of extended liability makes the investor responsible for managing full compensation of victims and partial liability of the investor is shifted onto the entrepreneur in the form of higher payments. The total costs associated with the environmental accident are jointly internalized by the lending institution and the entrepreneur. The role of such a regulation as an instrument of reallocation of the costs of risk is thus highlighted.

The paper is organized as follows. In section 2, we present the two-period CSV model. The problem that has to be solved is stated in section 3. The optimal contract under extended liability to lenders is characterized in section 4. In section 5, we evaluate the impact of a financial risk associated with an environmental accident on loan conditions. We conclude in section 6. All proofs are gathered in the appendix.

2 The financial contracting problem

We consider a risk-neutral entrepreneur who wants to invest in a project that costs I. Given that he has no initial wealth, the money needed is raised externally from outside investors.

The project is risky with two respects:

- It yields random cash-flows (revenues net of production costs, but gross of financial costs) at date $t = 1$ and date $t = 2$. These cash-flows, denoted $\tilde{y}_t$, for $t = 1; 2$, are supposed to be random variables distributed on the support $[\underline{y}; \overline{y}]$, according to a continuous distribution function F_t with positive density function f_t, where $\underline{y} > 0$ and $\overline{y}$ is considered as very large.

  For simplicity, we assume that $\tilde{y}_1$ and $\tilde{y}_2$ are independent. And the hazard rate for $\tilde{y}_2$, $f_2/[1 - F_2]$ is increasing.

- And the entrepreneur's activity may cause an environmental damage resulting in a financial risk: a loss $\tilde{d} = d$ can occur in the second period with probability p, $0 < p < 1$, ($\tilde{d} = 0$ with probability $(1 - p)$). This loss may correspond to damages to victims. We suppose that $\underline{y} < d < 2\underline{y}$.

These two types of financial risks are assumed independent. Under a liability rule, the cost of the environmental risk is imposed on the entrepreneur who must pay for it. We assume that this loss is deduced from the project cash-flows, which amounts to considering that there is a priority of victims' compensation over payments to the lending institution. The assumption on the level of d means that a one-period cash-flows may not be sufficient to cover the loss. However, over the two periods the firm has enough cash-flows to completely compensate the victims (however, he may default on the bank). The amount of damages, d, is assumed to be known to both the entrepreneur and the investor without any cost.[5] This perfect observation of the amount of damages leads us to think of d as some recovering costs that the entrepreneur-polluter would have to pay directly to the victims or to some institution that has dealt with emergency cleanup efforts and seeks to recover damages (exactly as under CERCLA).[6] Finally, we assume that the project is worth undertaking: $E[y] - pd > I$.

The set up of the moral hazard problem is the costly state verification approach. The realization of $\tilde{y}_t$ is learned at date t, $t = 1, 2$, by the entrepreneur but not by the investor, i.e., there is asymmetric information at date t, $t = 1, 2$. However, we suppose that the investor can verify the true cash-flows at a cost that is a function of the value of the entrepreneur's outcome. We denote $c_t()$ the verification or inspection cost function at date t, and we suppose that c_t is a non-decreasing function of the amount to be inspected.

In order to limit the complexity of the problem, it is assumed that the firm cannot borrow again after the realization of date one cash-flow $\tilde{y}_1$ and that no dividends can be distributed to the entrepreneur before all the obligations to the selected investor as well as to the potential victims are fulfilled.

[5] With this assumption of perfect observation, this model constitutes a particular simple case of the model of Caillaud et al. (2000). However, the dynamic context adds a degree of complexity.

[6] CERCLA created a Hazardous Substance Response fund (financed with an ad valorem tax) for dealing with emergency cleanup efforts. Once the emergency has been dealt with, the government seeks to recover damages from the liable party or parties.

A competitive environment with respect to the number of potential investors is considered: we suppose that there exists a large number of potential investors that may write contracts specifying decisions of cash-flows inspection at each date and reimbursement plans contingent on the reported cash-flows. This assumption implies that the optimal contract is the contract that maximizes the entrepreneur's expected payoffs, subject to the constraint that the selected investor makes zero-expected profits. By assumption, each potential investor is risk neutral.

In this dynamic context, we consider the highest degree of commitment namely full commitment under which the contract signed between the two parties covers the duration of the relationship and cannot be broken or renegotiated. Under full commitment, the Revelation Principle applies: since the contract cannot be redefined, the parties interact only once. Hence, we focus our attention on direct incentive-compatible contracts (or mechanisms) that is on contracts in which the entrepreneur makes truthful cash-flows reports at dates 1 and 2. Depending on what the entrepreneur reports, and on the occurrence (or not) of an environmental damage, the contract then specifies for each date whether inspection should occur or not and how much the entrepreneur must pay to the investor.

The timing with which events take place is the following. At date zero, the contract is signed and \$$I$ is invested. At date one, the first period cash-flow is realized and the firm reports y_1; based on y_1, inspection takes place with probability $\delta_1(y_1)$ and the first period payment is made to the investor according to the contract terms. We denote $t_1(y_1)$ the first period payment when inspection takes place at date 1 and $T_1(y_1)$ the first period payment when no inspection takes place at date 1. During period two, an environmental damage may occur giving place to a loss, the amount of which, $\tilde{d}$, $\tilde{d} \in \{0; d\}$, is observable without any cost by both the entrepreneur and the investor. At date two, the second period cash-flow is realized and the firm reports y_2. We denote $\delta_2(y_1; y_2; \tilde{d})$ the probability that inspection takes place at date 2. Similarly, we denote $t_2(y_1; y_2; \tilde{d})$ (resp. $T_2(y_1; y_2; \tilde{d})$) the second period payment to the investor when no inspection takes place (resp. does not take place) at date 1.[7]

A contract with a deterministic auditing technology then specifies:

- an inspection function at date 1, $\delta_1 : \mathbb{R}_+ \to \{0; 1\}$, and an inspection

[7] Note that $t_2(y_1; y_2; \tilde{d})$ and $T_2(y_1; y_2; \tilde{d})$ will take different values whether inspection takes place at date 2 or not.

function at date 2, $\delta_2 : \mathbb{R}_+ \times \mathbb{R}_+ \times \{0, d\} \to \{0, 1\}$;

- payments at date 1, $t_1 : \mathbb{R}_+ \to \mathbb{R}$ and $T_1 : \mathbb{R}_+ \to \mathbb{R}$, and payments at date 2, $t_2 : \mathbb{R}_+ \times \mathbb{R}_+ \times \{0, d\} \to \mathbb{R}$ and $T_2 : \mathbb{R}_+ \times \mathbb{R}_+ \times \{0, d\} \to \mathbb{R}$.

Observe that since in the event of an environmental accident, the loss associated in monetary terms is observable by both parties without any cost, we are in a situation where there is private information only on cash-flows, exactly as in models such as those of Townsend (1979) and Gale and Hellwig (1985). Consequently, we only have to consider revelation constraints with respect to final cash-flows reports.

3 Statement of the problem

Given the timing of events and the assumptions on available information, we start by determining the form of the contract at date 2.

3.1 The second-period problem

Since date two is the last date and since we only have to consider truth-telling constraints with respect to assets reports, for a given y_1, the results of Gale and Hellwig (1985) apply for each state of nature (loss and no-loss). In other words, the second-period optimal contract is a debt contract contingent on the realized loss associated with an environmental accident. This debt contract is characterized by the face value of the debt (the amount to be reimbursed at date two) that depends on the magnitude of the loss. Let denote $\hat{T}_2(y_1; \tilde{d})$ (resp. $\hat{t}_2(y_1; \tilde{d})$) the face value of the debt that is the optimal payment at date 2 when no inspection takes place at date 1 (resp. when inspection takes place at date 1).[8] We have the following proposition :

Proposition 1 For an optimal contract,

(i) $\delta_2(y_1; y_2; \tilde{d}) = 1 \iff (y_1 - T_1(y_1)) + y_2 - \tilde{d} < \hat{T}_2(y_1; \tilde{d}); \ \tilde{d} \in \{0; d\}$

(ii) $T_2(y_1; y_2; \tilde{d}) = \begin{cases} \hat{T}_2(y_1; \tilde{d}); \ \tilde{d} \in \{0; d\}; & \text{if } \delta_2(y_1; y_2; \tilde{d}) = 0 \\ y_2 + y_1 - T_1(y_1) - \tilde{d}, \ \tilde{d} \in \{0; d\}; & \text{if } \delta_2(y_1; y_2; \tilde{d}) = 1 \end{cases}$

[8] If a contract is optimal, the payment schedule at date 2 when no inspection takes place at that date is independent of the reported value of cash-flows, y_2.

(iii) $\bar{T}_2(y_1; d) = \bar{T}_2(y_1; 0) - d$.

(iv) The same results hold when inspection takes place at date 1: $\left(T_1(y_1); T_2(y_1; y_2; \tilde{d}); \bar{T}_2(y_1; \tilde{d})\right)$ are then replaced by $\left(t_1(y_1); t_2(y_1; y_2; \tilde{d}); \bar{t}_2(y_1; \tilde{d})\right)$.

Proof. See appendix.

Point (i) of proposition 1 establishes that inspection takes place at date two if and only if the entrepreneur does not have sufficient earnings left to meet the required date two payment to the lending institution. By point (ii), when inspection takes place at date two, the firm is declared bankrupt : the investor confiscates an amount corresponding to the earnings net of compensation costs and the entrepreneur ceases its activity. When inspection does not take place at date 2, the firm pays a fixed (independent of y_2) amount , $\bar{T}_2(y_1; \tilde{d})$ for $\tilde{d} \in \{0; d\}$. Point (iii) establishes that the fixed amount to be paid in the no-loss state of nature, $\bar{T}_2(y_1; 0)$, is greater than the amount that has to be paid in the loss state of nature, $\bar{T}_2(y_1; d)$. Hence, by virtue of (i) to (iii), the second period contract, viewed as a one period contract, can be interpreted as follows. The initial contract is a debt contract with face-value $\bar{T}_2(y_1; 0)$. After an environmental accident, the face value of the debt is reduced according to the amount of damages. This means that full insurance is provided by the lending institution to the entrepreneur. This reduction of the face value of the debt is motivated by the existence of transaction costs (costly auditing). By reducing the face value of the debt when an environmental accident occurs, the investor reduces the risk of default of the entrepreneur in the debt contract which in turn reduces transaction costs. As highlighted by Caillaud et al. (2000), "corporate demand for insurance emerges jointly with financial contracting decisions"(p. 78).

Observe, from point (i), that when inspection takes place at date 2, that is when the firm is declared bankrupt, and $\tilde{d} = d$, as $\underline{y} < d$ by assumption, the amount $y_2 + y_1 - T_1(y_1) - d$ may not be positive. And if $y_2 + y_1 - T_1(y_1) \leq d$, i.e., if over the two periods, the entrepreneur does not have sufficient earnings left to fully compensate the victims, as we consider that investors are subject to a rule of extended liability, they must accept a negative payment (which allows full compensation of victims). Finally, given the form of the second period contract, the two-period contract with deterministic auditing now reduces to ($\delta_1(y_1)$, $t_1(y_1)$, $T_1(y_1)$, $\bar{t}_2(y_1; \tilde{d})$, $\bar{T}_2(y_1; \tilde{d})$) we seek to characterize.

3.2 The entrepreneur's optimization problem

Given our informational problem, some incentives constraints have to be considered as well as the participation constraint for the lending institution that prevails under a rule of extended liability.

3.2.1 The wealth constraints and the incentive constraints

Given that the firm has no equity and that it cannot have recourse to other investors after date 0, some wealth constraints on first period payments have to be considered. The date one payments cannot exceed the true realized cash-flow. Formally,

$$\begin{array}{l} t_1(y_1) \leq y_1 \\ T_1(y_1) \leq y_1. \end{array} \tag{1}$$

Limiting ourself to truthful revelation mechanisms, the truth-telling constraints have to be identified. Given the form of the second-period contract, we only have to consider truth-telling constraints with respect to the first period cash-flow. In addition, we need only worry about announcements for which inspection does not take place at date 1, i.e., announcements of y_1 for which $\delta_1(y_1) = 0$.[9] Consequently, the number of incentive-compatible constraints is reduced to two. Also, account must be taken of the fact that the wealth constraints delimitate the set of possible lies.

Let x_1 and y_1 two realizations of $\tilde{y}_1$ for which inspection does not take place, i.e., $\delta_1(x_1) = \delta_1(y_1) = 0$, and suppose that x_1 satisfies $T_1(x_1) \leq y_1$. The incentive-compatible constraint that the entrepreneur with y_1 does not want to announce x_1 is :

$$T_1(y_1) + E_{\tilde{e}}\tilde{T}_2(y_1; \tilde{e}) \leq T_1(x_1) + E_{\tilde{e}}\tilde{T}_2(x_1; \tilde{e}) \tag{2}$$

Note that $T_1(y_1) + E_{\tilde{e}}\tilde{T}_2(.; \tilde{e})$, which can be rewritten as $E_{\tilde{e}}\left[T_1(y_1) + \tilde{T}_2(y_1; \tilde{e})\right]$, represents the entrepreneur's expected total payment (expectation taken with respect to the loss associated with the environmental accident). Given that for $y_1 \geq x_1$, $y_1 \geq x_1 \geq T_1(x_1)$, we deduce from (2) that the expected total payment cannot increase in the no-inspection region.

Let x_1 and y_1 two realizations of $\tilde{y}_1$ for which $\delta_1(x_1) = 0$ and $\delta_1(y_1) = 1$, and suppose that x_1 satisfies $T_1(x_1) \leq y_1$. In this case, the incentive-

[9] If a misrepresentation leads to inspection, the lie is discovered and first-period cash-flows are taken by the lending institution.

compatible constraint according to which the entrepreneur with y_1 has nothing to gain by announcing x_1 is :

$$t_1(y_1) + E_{\tilde{\theta}}\hat{t}_2(y_1;\tilde{\theta}) \le T_1(x_1) + E_{\tilde{\theta}}\hat{T}_2(x_1;\tilde{\theta}) \tag{3}$$

(3) implies that the expected total payment is greater in the no-inspection region.

3.2.2 The participation constraint of the lending institution

The participation constraint of the lending institution is affected by the fact that it may be held liable. A regulation of extended liability to lenders imposes some compensatory costs on the investor when an accident does occur: in particular, in the case where $\underline{y} \le y_2 \le \tilde{\theta} - (y_1 - T_1)$, under such a regulation, the investor appropriates the earnings left and agrees to receive a negative payment (victims are then fully compensated). The participation constraint thus writes

$$\begin{aligned}
&\int_{\underline{y}}^{\bar{y}} \delta_1(y_1)\Bigg\{ t_1(y_1) - c_1(y_1) + E_{\tilde{\theta}}\Bigg[\int_{\underline{y}}^{\hat{t}_2(y_1;\tilde{\theta})+\tilde{\theta}-(y_1-t_1)} [Y_2^I - \tilde{\theta} - c_2(Y_2^I)]dF_2 \\
&\quad + \int_{\hat{t}_2(y_1;\tilde{\theta})-(y_1-t_1)}^{\bar{y}} \hat{t}_2(y_1;\tilde{\theta})dF_2\Bigg]\Bigg\} dF_1 + \int_{\underline{y}}^{\bar{y}} [1-\delta_1(y_1)]\Bigg\{T_1(y_1) \\
&+ E_{\tilde{\theta}}\Bigg[\int_{\underline{y}}^{\hat{T}_2(y_1;\tilde{\theta})+\tilde{\theta}-(y_1-T_1)} [Y_2 - \tilde{\theta} - c_2(Y_2)]dF_2 + \int_{\hat{T}_2(y_1;\tilde{\theta})-(y_1-T_1)}^{\bar{y}} \hat{T}_2(y_1;\tilde{\theta})dF_2\Bigg]\Bigg\} dF_1 = I
\end{aligned} \tag{4}$$

where $Y_2^I \equiv y_2 + y_1 - t_1(y_1)$ and $Y_2 \equiv y_2 + y_1 - T_1(y_1)$. In summary, a feasible contract $(\delta_1(y_1), t_1(y_1), T_1(y_1), \hat{t}_2(y_1;\tilde{\theta}), \hat{T}_2(y_1;\tilde{\theta}))$ is a contract that satisfies the wealth constraints (1), the incentive-compatible constraints (2) and (3), and the participation constraint (4).

3.2.3 The optimization problem

To characterize the optimal feasible financial contract, we need to determine $(\delta_1(y_1), t_1(y_1), T_1(y_1), \hat{t}_2(y_1;\tilde{\theta}), \hat{T}_2(y_1;\tilde{\theta}))$ that solve the problem of maximizing the entrepreneur's expected payoffs subject to constraints (1), (2), (3) and (4). Given risk neutrality of both parties, this problem is equivalent to the problem of minimizing the expected inspection costs subject to the same constraints. We thus need to determine $(\delta_1(y_1); t_1(y_1); T_1(y_1); \hat{t}_2(y_1;\tilde{d}); \hat{T}_2(y_1;\tilde{d}))$ that solve the following problem:

$$\text{Min} \int_{\underline{y}}^{\bar{y}} \delta_1(y_1) \left\{ c_1(y_1) + E_{\tilde{d}} \left[\int_{\underline{y}}^{\hat{t}_2(y_1;\tilde{d})+\tilde{d}-(y_1-t_1)} c_2(Y_2^{\omega}) dF_2 \right] \right\} dF_1$$

$$+ \int_{\underline{y}}^{\bar{y}} [1-\delta_1(y_1)] \left\{ E_{\tilde{d}} \left[\int_{\underline{y}}^{\hat{T}_2(y_1;\tilde{d})+\tilde{d}-(y_1-T_1)} c_2(Y_2) dF_2 \right] \right\} dF_1 \tag{5}$$

under constraints (1), (2), (3), (4) and $\delta_1 = 0$ or 1, where $Y_2^{\omega} \equiv y_2 + y_1 - t_1(y_1)$ and $Y_2 \equiv y_2 + y_1 - T_1(y_1)$.

The contract form and its possible interpretations are given in the next section.

4 Optimal financial contract

Under a rule of extended liability, the selected investor becomes in charge of forcing the entrepreneur to fully internalize the costs of the accident. The investor will manage this by setting the financial contract. More precisely, the following results can be derived:

Proposition 2 The optimal contract under extended liability is such that:

(i) $\hat{t}_2(y_1;\tilde{d}) - y_1 + t_1 = \kappa - \tilde{d}$, for $\tilde{d} \in \{0; d\}$.

Moreover, if c_2 is strictly increasing, $t_1 = y_1$ and $\hat{t}_2(y_1;\tilde{d}) = \kappa - \tilde{d}$, for $\tilde{d} \in \{0; d\}$.

(ii) $T_1(y_1) + \hat{T}_2(;\tilde{d}) = \rho(d) - \tilde{d}$, for $\tilde{d} \in \{0; d\}$; where $\rho(d) = \rho + pd$

(iii) $\delta_1 = 1 \iff y_1 \leq \eta$; where $\eta \in [\underline{y}; \bar{y}]$

Proof. See appendix.

The optimal contract with extended liability is characterized by the triplet $(\kappa; \eta; \rho(d))$, where η denotes the fixed date 1 payment, κ is the final payment when inspection takes place at date 1 (or equivalently the probability of bankruptcy at date 2 when inspection takes place at date 1), and $\rho(d)$ is the fixed amount that has to be paid over the two period (or call price). With these three components, this contract is similar to the one derived by Chang, and it can be interpreted as a bond contract. But the present contract has the particularity to be contingent upon the occurrence of an environmental accident: it specifies that when an accident does occur, the payments are reduced by the amount of the loss. Again, this means that full insurance is provided by the lending institution to the entrepreneur because of risk aversion towards transaction costs from the investor.

Inspection takes place at date one if and only if the first period cash-flow is not sufficient (smaller than the required payment). Observe that in this dynamic context, first-period inspection is not associated with the termination of the firm's activity.

Finally, if c_2 is strictly increasing, it is optimal to set $T_1(y_1) = y_1$ for y_1 satisfying $y_1 < \nu(d)$. And if $y_1 > \nu(d)$, there is no default risk at date 2 and $T_1(y_1)$ can be less than y_1.

The contingent contract with extended liability is represented in the following figures, under the assumption that c_2 is strictly increasing.

Insert figures 1a) and 1b)

In figures 1a) and b), both period payments are represented in terms of y_1. Observe, in figure 1a) that in case the entrepreneur receives a high payoff in period 1, he can on a voluntary basis pay the total amount, $\nu + pd$, at the end of the first period. The total amount is then reimbursed by anticipation, and if an environmental accident occurs during period 2, the investor pays back the entrepreneur the amount of damages, d, at date 2.

Insert figures 2a) and 2b)

In figures 2a) and b), second period payments are represented in terms of y_2, under both cases of $\delta_1 = 1$ and $\delta_1 = 0$. These are typical representation of debt contracts, except the straight line appearing in the negative orthant which is due to the assumption of extended liability.

An alternative interpretation of the optimal contract contingent on the occurrence of an accident involving a compulsory insurance covenant can be given. More precisely, this contract can be imitated by a non-contingent financial contract plus an insurance policy. This can be highlighted writing the entrepreneur's ex post cash-flows when an environmental accident occurs. For example, in the case where inspection has taken place at date 1 but not at date 2, these cash-flows are:

$$y_1 - y_1 + y_2 - d - (\kappa - d)$$

They may be written as

$$y_1 - y_1 + y_2 - d + d - \kappa$$

It is as if the entrepreneur had received a coverage of d. The insurance contract is characterized by a coverage of d and its price, π^*, corresponds to the fair premium which amounts to

$$\pi^* = p \int_{\underline{y}}^{\eta} \left[\int_{\underline{y}}^{d} d\, dF_2 + \int_{d}^{\overline{y}} d\, dF_2 \right] dF_1 + p \int_{\eta}^{\nu(d)} \left[\int_{\underline{y}}^{d} d\, dF_2 + \int_{d}^{\overline{y}} d\, dF_2 \right] dF_1 + p \int_{\nu(d)}^{\overline{y}} d\, dF_1$$

that is

$$\pi^* = pd$$

The premium corresponds to the expected compensation costs when compensatory payments are imposed on the lender through a liability rule, that is expected loss. The lending institution will propose to lend an amount equal to the sum of the required investment I and of the premium π^* under a bond contract characterized by the triplet $(\eta; \kappa; \nu(d))$. The bond contract will include a compulsory covenant requiring that the entrepreneur subscribes to an insurance policy as characterized above. The financial risk associated with the environmental accident entails an additional covenant of compulsory insurance to the contract. In the spirit of Chang (1990), the optimal two-period financial contract can be considered as a bond contract which includes four covenants: a covenant which directly restrict the payment of dividends and a covenant restricting subsequent financing policy, both taken as hypothesis, a covenant specifying a bonding activity expenditure namely the required purchase of insurance and a covenant modifying the pattern of payoffs to bondholders (sinking fund and callability provisions), both emerging at the optimum.[10]

Moreover, with respect to the contract characterized by Chang, we have that an additional financial risk associated with an environmental accident increases the financing: the optimal financial contract covers a lent amount of $I + pd$ whereas in the absence of environmental risk (which would correspond to the case $p = 0$) the optimal contract would cover a lent amount of I.

Finally, the lending institution as well as the entrepreneur are ex-ante indifferent between the non-contingent financial contract that includes an

[10] For a description of various kind of bond covenants and their impact on bondholder-stockholder conflict, see C.W. Smith and J.B. Warner (1979).

insurance covenant and the contingent financial contract. Indeed, both contracts yield zero-expected profits to the lending institution. And the expected payments made by the entrepreneur amount to the same.

Under a rule of extended liability, the costs associated with the environmental accident are fully internalized. In the next section, we show how these costs are jointly internalized by the investor and the entrepreneur.

5 Impact of environmental risks on loan conditions

As an additional financial risk does not affect the contract form (with respect to a situation where there would be only one source of financial risk) but only increases the financing (for the entrepreneur to purchase insurance), we can evaluate the impact of an additional financial risk on loan conditions by proceeding to a comparative static analysis on the level of the amount lent. Such an analysis leads to the following results:

Proposition 3

(i) $\frac{\partial \kappa}{\partial I(d)} > 0$

(ii) $\frac{\partial \eta}{\partial I(d)} > 0$

(iii) $\frac{\partial \gamma(d)}{\partial I(d)} > 0$

Proof. See appendix.

The effects of environmental risks on loan conditions under a regulation of extended liability may be summarized as follows :

- it increases the probability of bankruptcy at date 2;
- it increases the inspection region at date 1 (the fixed date 1 payment from the entrepreneur to the lending institution is increased);
- it increases the total payment towards the financial institution over the two periods.

Liability for a …nancial risk associated with environmental matters leads the lending institution to strengthen loan conditions. The investor's partial liability is shifted onto the entrepreneur in the form of higher …xed date 1 payment and higher total payment over the two periods. But it also implies a higher probability of bankruptcy at date 2.

Subject to a regulation of extended liability, the lending institution uses its contractual relationship with the entrepreneur to implement full compensation of the victims without having to draw from its capital. Indeed, this regulation does not a¤ect the investor's payo¤: it has zero expected pro…ts (the participation constraint of the lending institution binds). Costs associated with environmental damages are jointly internalized by the investor and the entrepreneur. Through the design of the …nancial contract, a transfer of surplus from the entity investor/entrepreneur to the victims is implemented.

6 Concluding remarks

This paper provides an analysis of dynamic …nancial contracts when the borrower may face two types of …nancial risks one of which is related to an environmental damage. The contract form is derived under a liability rule allowing for extended liability to lenders. The optimal …nancial contract consists in a contingent bond contract, i.e., a bond contract that is dependent upon the realization of an environmental accident. Because of existing transaction costs, full insurance is provided by the lending institution to the entrepreneur. The contingent contract can be replicated with a non-contingent bond contract incorporating, among other covenants, a compulsory insurance covenant. And the possibility of an environmental accident increases the …nancing (for the entrepreneur to purchase insurance) with respect to a situation where this …nancial risk is not taken into account.

Under extended liability, the total costs associated with the environmental accident are jointly internalized by the investor and the entrepreneur. Indeed, a regulation of extended liability that allows victims' full compensation a¤ects the pattern of payments from the entrepreneur to the lending institution: the cost of debt capital is increased. This result is consistent with the strengthening of loan conditions observed in the United States. And the risk that the entrepreneur defaults on the bank is also increased. Dionne and Spaeter (1998) reach the same result in a di¤erent context: they consider a static model where a …rm is …nanced by equity and debt in order to invest in production and in a risk-reducing activity.

For a certain category of environmental well de…ned risks namely those associated with "small" damages[11], insurance coupled with a rule of lenders' partial liability appears to be the solution to achieve the goal of full victims' compensation. This "e¢ciency" result may be related to the result obtained by Pitchford (1995) who considers a moral hazard problem and who arrives to the conclusion that a "partial" liability rule would minimize the probability of an accident and thus enhance e¢ciency. Boyer and La¤ont (1997) also favor a partial lender liability rule with respect to risk reduction.

However, in this paper, attention was focused on the impact of a regulation of extended liability, leading to optimal ex-post compensation under ex-post moral-hazard, on the structure of the …nancial contract. Many aspects are not taken into account and in particular the incentive e¤ects of extended liability. Future research will be motivated by the more challenging question: does a system of extended liability achieving an optimal ex-post compensation leads also to an optimal ex-ante risk reduction?

[11] Remind the assumption on the level of d. This assumption on the cost of environmental damage may not sound that strange: as H. Smets (1992) underlines, the compensation costs of accidental pollutions are not that important for the state, the industry and insurers. In particular, these costs are far less important than the costs of pollution prevention of an industry.

Appendix

A Proof of proposition 1

(i) and (ii): See Gale et Hellwig (1985) and Caillaud, Dionne and Jullien (2000).

The proof of point (iii) is established considering that no inspection occurs at date 1. The result also applies if we consider that inspection takes place at date 1. The optimal values of $\rho_2(y_1;0)$ and $\rho_2(y_1;d)$ can be found solving the following program:

$$\underset{\rho_2(y_1;0);\rho_2(y_1;d)}{\text{Max}} \; -E_\theta\left[\int_{\underline{y}}^{\rho_2(y_1;\theta)+\theta-(y_1-T_1)} c_2(Y_2)dF_2\right]$$

subject to

$$E_\theta\left[\int_{\underline{y}}^{\rho_2(y_1;\theta)+\theta-(y_1-T_1)}\left[Y_2-\theta-c_2(Y_2)\right]dF_2+\int_{\rho_2(y_1;\theta)+\theta-(y_1-T_1)}^{\bar{y}}\rho_2(y_1;\theta)dF_2\right]=I$$

where $Y_2 \equiv y_2 + (y_1 - T_1)$.

Let L denote the Lagrangian of the program and λ the multiplier associated to the zero-expected profits constraint. Combining the first-order conditions $\frac{\partial L}{\partial \rho_2(\cdot;0)} = 0$ and $\frac{\partial L}{\partial \rho_2(\cdot;d)} = 0$, we obtain that $\rho_2(y_1;d) + d = \rho_2(y_1;0)$ at the optimum: the sum of the face-value of the debt and of losses is equalized across states. ■

B Proof of proposition 2

Consider problem (5) and transform it into an optimal control problem. A differential version of equation (2) is given by:

$$E_\theta\left(T_1' + \rho_2'(\theta)\right) \le 0 \qquad (6)$$

Let define the control variables $u_1 = T_1'$, $u_2 = \rho_2'(0)$, and $u_3 = \rho_2'(d)$.

Dropping the incentive constraint (3),[12] the problem we now have to solve is the following:

$$\text{Min} \int_{\underline{y}}^{\bar{y}} \delta_1(y_1)\left\{c_1(y_1) + E_\theta \int_{\underline{y}}^{\rho_2(y_1;\theta)+\theta-(y_1-t_1)} c_2(Y_2^I)dF_2\right\}dF_1$$

[12] The proof established by Chang applies in our context.

$$+\int_{\underline{y}}^{\bar{y}} [1-\delta_1(y_1)] \left[E_{\tilde{\varepsilon}} \int_{\underline{y}}^{\hat{T}_2(y_1;\tilde{\varepsilon})+\tilde{\varepsilon}-(y_1-T_1)} c_2(Y_2)dF_2 \right] dF_1$$

subject to

$$u_1 = T_1'$$

$$u_2 = \hat{T}_2'(0)$$

$$u_3 = \hat{T}_2'(d)$$

$$(1-p)(u_1+u_2) + p(u_1+u_3) \le 0$$

$$t_1 \le y_1$$

$$T_1 \le y_1$$

and the participation constraint for the lending institution (eq. 4).

In this optimization problem, the control variables are u_1, u_2 and u_3, the state variables are T_1, $\hat{T}_2(y_1;0)$ and $\hat{T}_2(y_1;d)$, and we denote μ_1, μ_2 and μ_3 the costate variables for respectively T_1, $\hat{T}_2(y_1;0)$ and $\hat{T}_2(y_1;d)$. Note that this control problem entails both constraints on control variables and state variables. The Hamiltonian is

$$H(T_1;\hat{T}_2(y_1;0);\hat{T}_2(y_1;d);u_1;u_2;u_3;\mu_1;\mu_2;\mu_3;y_1) =$$

$$\delta_1(y_1)\left\{ c_1(y_1) + E_{\tilde{\varepsilon}} \int_{\underline{y}}^{\hat{T}_2(y_1;\tilde{\varepsilon})+\tilde{\varepsilon}-(y_1-t_1)} c_2(Y_2^l)dF_2 \right\} f_1$$

$$+[1-\delta_1(y_1)] \left\{ E_{\tilde{\varepsilon}} \int_{\underline{y}}^{\hat{T}_2(y_1;\tilde{\varepsilon})+\tilde{\varepsilon}-(y_1-T_1)} c_2(Y_2)dF_2 \right\} f_1 + [1-\delta_1(y_1)][\mu_1 u_1 + \mu_2 u_2 + \mu_3 u_3] f_1$$

and the Lagrangian writes, after gathering of terms:

$$L = \delta_1 \left\{ c_1(y_1) - \beta(t_1 - c_1(y_1)) + E_{\tilde{\varepsilon}} \int_{\underline{y}}^{\hat{T}_2(y_1;\tilde{\varepsilon})+\tilde{\varepsilon}-(y_1-t_1)} c_2(Y_2^l)dF_2 \right.$$

$$- \beta \left[E_{\tilde{\varepsilon}} \int_{\underline{y}}^{\hat{T}_2(y_1;\tilde{\varepsilon})+\tilde{\varepsilon}-(y_1-t_1)} \left[Y_2^l - \tilde{\varepsilon} - c_2(Y_2^l) \right] dF_2 + E_{\tilde{\varepsilon}} \int_{\hat{T}_2(y_1;\tilde{\varepsilon})+\tilde{\varepsilon}-(y_1-t_1)}^{\bar{y}} \hat{t}_2(y_1;\tilde{\varepsilon})dF_2 \right]$$

$$\left. + \lambda_1(t_1 - y_1) \right\} f_1$$

$$+(1-\delta_1)\left\{ -\beta T_1 + E_{\tilde{\varepsilon}} \int_{\underline{y}}^{\hat{T}_2(y_1;\tilde{\varepsilon})+\tilde{\varepsilon}-(y_1-T_1)} c_2(Y_2)dF_2 \right.$$

$$-\beta\left[E_{\tilde d}\int_{\underline{y}}^{\hat T_2(y_1;\tilde d)+\tilde d-(y_1-T_1)}\left[Y_2-\tilde d-c_2(Y_2)\right]dF_2+E_{\tilde d}\int_{\hat T_2(y_1;\tilde d)+\tilde d-(y_1-T_1)}^{\bar y}\hat T_2(y_1;\tilde d)\,dF_2\right]$$

$$+\lambda_1(T_1-y_1)+\gamma\left[(1-p)(u_1+u_2)+p(u_1+u_3)\right]+\mu_1u_1+\mu_2u_2+\mu_3u_3\} f_1+\beta l$$

where γ, $\bar\lambda_1$, λ_1 and β are the multipliers associated to the respective constraints. First-order conditions for the maximization of the Lagrangian with respect to u_1, u_2 and u_3 are

$$\frac{\partial L}{\partial u_1}=\gamma+\mu_1=0$$

$$\frac{\partial L}{\partial u_2}=\gamma(1-p)+\mu_2=0$$

$$\frac{\partial L}{\partial u_3}=\gamma p+\mu_3=0$$

The other necessary conditions are

1. the state equations

$$T_1'=u_1$$
$$\hat T_2'(0)=u_2$$
$$\hat T_2'(d)=u_3$$

2. the costate equations

$$\mu_1'=-\frac{\partial L}{\partial T_1}$$
$$\mu_2'=-\frac{\partial L}{\partial \hat T_2(y_1;0)}$$
$$\mu_3'=-\frac{\partial L}{\partial \hat T_2(y_1;d)}$$

3. complementarity slackness

$$\gamma\ \geq 0;\quad (1-p)(u_1+u_2)+p(u_1+u_3)\ \leq 0;\quad \gamma\left[(1-p)(u_1+u_2)+p(u_1+u_3)\right]=0$$

$$\bar\lambda_1\geq 0;\quad t_1-y_1\leq 0;\quad \bar\lambda_1(t_1-y_1)=0$$

$$\lambda_1\geq 0;\quad T_1-y_1\leq 0;\quad \lambda_1(T_1-y_1)=0$$

$$\beta\geq 0;\quad (4)\geq 0;\quad -\beta(4)=0$$

where

$$\frac{\partial L}{\partial T_1} = \Big\{ (1+\beta)E_{\tilde d}\Big[c_2\big(\hat T_2(y_1;\tilde d)+\tilde d\big) f_2\big(\hat T_2(y_1;\tilde d)+\tilde d - y_1 + T_1\big)\Big]$$

$$- (1+\beta)E_{\tilde d}\int_{\underline{y}}^{\hat T_2(y_1;\tilde d)+\tilde d - y_1+T_1} c_2'(Y_2)dF_2$$

$$- \beta E_{\tilde d}\Big[1 - F_2\big(\hat T_2(y_1;\tilde d)+\tilde d - y_1 + T_1\big)\Big] + \lambda_1\Big\} f_1 \tag{7}$$

$$\frac{\partial L}{\partial \hat T_2(y_1;0)} = \Big\{(1-p)\Big[(1+\beta)c_2\big(\hat T_2(y_1;0)\big) f_2\big(\hat T_2(y_1;0) - y_1 + T_1\big)$$

$$- \beta\Big[1 - F_2\big(\hat T_2(y_1;0) - y_1 + T_1\big)\Big]\Big]\Big\} f_1 \tag{8}$$

$$\frac{\partial L}{\partial \hat T_2(y_1;d)} = \Big\{\Big[p(1+\beta)c_2\big(\hat T_2(y_1;d)+d\big) f_2\big(\hat T_2(y_1;d)+d - y_1 + T_1\big)$$

$$- \beta\Big[1 - F\big(\hat T_2(y_1;d)+d - y_1 + T_1\big)\Big]\Big]\Big\} f_1 \tag{9}$$

Let consider the region where inspection takes place at date 1 i.e., the region for which $\delta_1(y_1) = 1$. First-order conditions for the maximization of the Lagrangian with respect to t_1, $\hat t_2(y_1;0)$ and $\hat t_2(y_1;d)$ reduce to

$$\Big\{(1+\beta)E_{\tilde d}\Big[c_2\big(\hat t_2(y_1;\tilde d)+\tilde d\big) f_2\big(\hat t_2(y_1;\tilde d)+\tilde d - y_1 + t_1\big)\Big]$$

$$- (1+\beta)E_{\tilde d}\int_{\underline{y}}^{\hat t_2(y_1;\tilde d)+\tilde d - y_1 + t_1} c_2'(Y_2^I)dF_2$$

$$- \beta E_{\tilde d}\Big[1 - F_2\big(\hat t_2(y_1;\tilde d)+\tilde d - y_1 + t_1\big)\Big] + \bar\lambda_1\Big\} f_1 = 0 \tag{10}$$

$$(1-p)\Big\{(1+\beta)c_2\big(\hat t_2(y_1;0)\big) f_2\big(\hat t_2(y_1;0) - y_1 + t_1\big) - \beta\Big[1 - F_2\big(\hat t_2(y_1;0) - y_1 + t_1\big)\Big]\Big\} f_1 = 0 \tag{11}$$

$$p\Big\{(1+\beta)c_2\big(\hat t_2(y_1;d)+d\big) f_2\big(\hat t_2(y_1;d)+d - y_1 + t_1\big)$$

$$- \beta\Big[1 - F_2\big(\hat t_2(y_1;d)+d - y_1 + t_1\big)\Big]\Big\} f_1 = 0 \tag{12}$$

Combining equations (11) and (12) gives

$$\hat t_2(y_1;0) = \hat t_2(y_1;d) + d \tag{13}$$

Substituting (11) and (12) into (10) and using (13), we obtain

$$(1+\beta)\int_{\underline{y}}^{\hat{t}_2(y_1;0)-y_1+t_1} c_2'(Y_2^l)dF_2 = \beta\lambda_1(y_1) \tag{14}$$

and

$$(1+\beta)\int_{\underline{y}}^{\hat{t}_2(y_1;d)+d-y_1+t_1} c_2'(Y_2^l)dF_2 = \beta\lambda_1(y_1). \tag{15}$$

(i) The reasoning developed by Chang (1990) can be applied for each state of nature (loss and no-loss).

(ii) Assume that equation (6) is not binding. Hence, $\gamma = 0$. Substituting back into the first order conditions for maximization of the Lagrangian with respect to u_1 and u_2, we have that $\mu_1 = \mu_2 = 0$ which in turn implies that $\mu_1' = \mu_2' = 0$. The costate equations now write exactly as equations (10), (11) and (12), with $t_1(y_1)$, $\hat{t}_2(y_1;0)$ and $\hat{t}_2(y_1;d)$ replaced by $T_1(y_1)$, $\hat{T}_2(y_1;0)$ and $\hat{T}_2(y_1;d)$. The reasoning of the proof of i) of proposition 3 now applies: $\hat{T}_2(y_1;\tilde{d}) + \tilde{d} + T_1(y_1) - y_1$ equals a constant for $\tilde{d} \in \{0;d\}$. Taking the expectation with respect to $\tilde{d}$, we have that $E_{\tilde{d}}\left(T_1(y_1) - y_1 + \hat{T}_2(y_1;\tilde{d}) + \tilde{d}\right)$ must also equal a constant. But this implies that $E_{\tilde{d}}\left(T_1' - 1 + \hat{T}_2'(\tilde{d})\right) = 0$ or equivalently that $E_{\tilde{d}}\left(T_1' + \hat{T}_2'(\tilde{d})\right) = 1$, a contradiction. Hence, equation (6) holds with equality which means that $E_{\tilde{d}}\left(T_1(y_1) + \hat{T}_2(y_1;\tilde{d})\right)$ equals a constant ν. The total liabilities (over the two period) in each state can be established straightforwardly using $\hat{T}_2(;0) = \hat{T}_2(;d) + d$, and the binding incentive-compatible constraint.

(iii) See Chang (1990).

■

C Proof of proposition 3

The program that has to be solved in order to completely characterize the optimal contingent financial contract with extended liability to lenders is the following

$$\max_{\kappa;\eta;\nu(d)} -\left[\int_{\underline{y}}^{\eta} c_1(y_1)dF_1 + \int_{\underline{y}}^{\eta}\int_{\underline{y}}^{\eta} c_2(y_2)dF_2dF_1 + \int_{\eta}^{\nu(d)}\int_{\underline{y}}^{\nu(d)-y_1} c_2(y_2)dF_2dF_1\right]$$

subject to

$$g(\kappa;\eta;\nu(d)) = 0$$

where

$$g(\kappa;\eta;\nu(d)) = \int_{\underline{y}}^{\eta}\left[(y_1 - c_1(y_1)) + \int_{\underline{y}}^{\kappa}(y_2 - c_2(y_2))dF_2 + \int_{\kappa}^{\overline{y}}\kappa\, dF_2\right]dF_1$$

$$+ \int_{\eta}^{\nu(d)}\left[y_1 + \int_{\underline{y}}^{\nu(d)-y_1}(y_2 - c_2(y_2))dF_2 + \int_{\nu(d)-y_1}^{\overline{y}}(\nu(d) - y_1)dF_2\right]dF_1$$

$$+ \int_{\nu(d)}^{\overline{y}}\nu(d)dF_1 - pd - I$$

The first-order conditions characterizing the optimal solution are

$$L_{\kappa} = -(1+\beta)c_2(\kappa)f_2(\kappa) + \beta[1 - F_2(\kappa)] = 0 \tag{16}$$

$$L_{\eta} = \left\{-(1+\beta)c_1(\eta) + (1+\beta)\int_{\kappa}^{\nu(d)-\eta}c_2(y_2)dF_2\right.$$

$$\left.+\beta\left[\kappa[1 - F_2(\kappa)] - (\nu(d) - \eta)[1 - F_2(\nu(d) - \eta)] - \int_{\kappa}^{\nu(d)-\eta}y_2 dF_2\right]\right\}f_1(\eta) = 0 \tag{17}$$

$$L_{\nu(d)} = -(1+\beta)\int_{\eta}^{\nu(d)}c_2(\nu(d) - y_1)f_2(\nu(d) - y_1)dF_1$$

$$+\beta\left[[1 - F_1(\nu(d))] + \int_{\eta}^{\nu(d)}[1 - F_2(\nu(d) - y_1)]dF_1\right] = 0 \tag{18}$$

$$L_{\beta} = g(\kappa;\eta;\nu(d)) = 0$$

Second order partial derivatives are

$$L_{\kappa\kappa} = -(1+\beta)\left(c_2'(\kappa)f_2(\kappa) + c_2(\kappa)f_2'(\kappa)\right) - \beta f_2(\kappa) < 0$$

$$L_{\kappa\eta} = L_{\eta\kappa} = 0$$

$$L_{\kappa\gamma(d)} = L_{\gamma(d)\kappa} = 0$$

$$L_{\eta\eta} = \left\{ -(1+\beta)c_1' - (1+\beta)c_2(\gamma(d)-\eta)f_2(\gamma(d)-\eta) + \beta[1 - F_2(\gamma(d)-\eta)] \right\} f_1(\eta) < 0$$

$$L_{\eta\gamma(d)} = \{(1+\beta)c_2(\gamma(d)-\eta)f_2(\gamma(d)-\eta) - \beta[1 - F_2(\gamma(d)-\eta)]\} f_1(\eta) > 0$$

$$L_{\gamma(d)\eta} = L_{\eta\gamma(d)} > 0$$

$$L_{\gamma(d)\gamma(d)} = -(1+\beta)\int_{\eta}^{\gamma(d)} c_2'(\gamma(d)-y_1)f_2(\gamma(d)-y_1)dF_1$$

$$-(1+\beta)\int_{\eta}^{\gamma(d)} c_2(\gamma(d)-y_1)f_2'(\gamma(d)-y_1)dF_1 - \beta\int_{\eta}^{\gamma(d)} f_2(\gamma(d)-y_1)dF_1 < 0$$

And the optimal solution indeed solves the maximisation problem if the successive principal minors of the Bordered Hessian evaluated at the optimal solution alternate in sign in the following way:

$$\begin{vmatrix} L_{\kappa\kappa} & L_{\kappa\eta} & \frac{\partial g}{\partial \kappa} \\ L_{\eta\kappa} & L_{\eta\eta} & \frac{\partial g}{\partial \eta} \\ \frac{\partial g}{\partial \kappa} & \frac{\partial g}{\partial \eta} & 0 \end{vmatrix} > 0; |H^*| = \begin{vmatrix} L_{\kappa\kappa} & L_{\kappa\eta} & L_{\kappa\gamma(d)} & \frac{\partial g}{\partial \kappa} \\ L_{\eta\kappa} & L_{\eta\eta} & L_{\eta\gamma(d)} & \frac{\partial g}{\partial \eta} \\ L_{\gamma(d)\kappa} & L_{\gamma(d)\eta} & L_{\gamma(d)\gamma(d)} & \frac{\partial g}{\partial \gamma(d)} \\ \frac{\partial g}{\partial \kappa} & \frac{\partial g}{\partial \eta} & \frac{\partial g}{\partial \gamma(d)} & 0 \end{vmatrix} < 0;$$

where

$$\frac{\partial g}{\partial \kappa} = \int_{\underline{y}}^{\eta} \left(-c_2(\kappa)f_2(\kappa) + 1 - F_2(\kappa)\right) dF_1$$

$$\frac{\partial g}{\partial \eta} = \left(-c_1(\eta)f_1(\eta) - \int_{\kappa}^{\gamma(d)-\eta} (y_2 - c_2(y_2))\, dF_2 + \int_{\kappa}^{\bar{y}} \kappa\, dF_2 - \int_{\gamma(d)-\eta}^{\bar{y}} (\gamma(d)-\eta) dF_2 \right) f_1(\eta)$$

$$\frac{\partial g}{\partial \gamma(d)} = \int_{\eta}^{\gamma(d)} \left(-c_2(\gamma(d)-y_1)f_2(\gamma(d)-y_1) + 1 - F_2(\gamma(d)-y_1)\right) dF_1 + 1 - F_1(\gamma(d))$$

Observe that $\frac{\partial g}{\partial \eta} < 0$ and that $\frac{\partial g}{\partial \kappa}$ and $\frac{\partial g}{\partial \gamma(d)}$ are positive under the assumption that $\bar{y}$ is very high.

It can be checked that the first principal minor is indeed positive: $\begin{vmatrix} L_{\kappa\kappa} & 0 & \frac{\partial g}{\partial \kappa} \\ 0 & L_{\eta\eta} & \frac{\partial g}{\partial \eta} \\ \frac{\partial g}{\partial \kappa} & \frac{\partial g}{\partial \eta} & 0 \end{vmatrix} = -\left(\frac{\partial g}{\partial \eta}\right)^2 L_{\kappa\kappa} - \left(\frac{\partial g}{\partial \kappa}\right)^2 L_{\eta\eta} > 0.$

With respect to $|H^*|$, $|H^*| = \begin{vmatrix} L_{\kappa\kappa} & 0 & 0 & \frac{\partial g}{\partial \kappa} \\ 0 & L_{\eta\eta} & L_{\eta\gamma(d)} & \frac{\partial g}{\partial \eta} \\ 0 & L_{\gamma(d)\eta} & L_{\gamma(d)\gamma(d)} & \frac{\partial g}{\partial \gamma(d)} \\ \frac{\partial g}{\partial \kappa} & \frac{\partial g}{\partial \eta} & \frac{\partial g}{\partial \gamma(d)} & 0 \end{vmatrix}$, expanding the determinant gives

$$\begin{aligned} |H^*| = {} & L_{\kappa\kappa}\frac{\partial g}{\partial \eta}\left(L_{\gamma(d)\eta}\frac{\partial g}{\partial \gamma(d)} - L_{\gamma(d)\gamma(d)}\frac{\partial g}{\partial \eta}\right) \\ & - L_{\kappa\kappa}\frac{\partial g}{\partial \gamma(d)}\left(L_{\eta\eta}\frac{\partial g}{\partial \gamma(d)} - L_{\eta\gamma(d)}\frac{\partial g}{\partial \eta}\right) \\ & - \left(\frac{\partial g}{\partial \kappa}\right)^2\left(L_{\eta\eta}L_{\gamma(d)\gamma(d)} - L_{\gamma(d)\eta}L_{\eta\gamma(d)}\right) \end{aligned}$$

and necessary conditions for $|H^*|$ to be negative are

$$\begin{aligned} & \left(L_{\gamma(d)\eta}\frac{\partial g}{\partial \gamma(d)} - L_{\gamma(d)\gamma(d))}\frac{\partial g}{\partial \eta}\right) < 0; \\ & \left(L_{\eta\eta}\frac{\partial g}{\partial \gamma(d)} - L_{\eta\gamma(d)}\frac{\partial g}{\partial \eta}\right) < 0; \\ & \left(L_{\eta\eta}L_{\gamma(d)\gamma(d)} - L_{\gamma(d)\eta}L_{\eta\gamma(d)}\right) > 0. \end{aligned}$$

Now the effects of a change in the amount borrowed on the characteristics of the financial contract can be evaluated, applying the standard method of comparative statics analysis of constrained optimization problem, as follows

$$\frac{\partial \kappa}{\partial l(d)} = \begin{vmatrix} 0 & 0 & 0 & \frac{\partial g}{\partial \kappa} \\ 0 & L_{\eta\eta} & L_{\eta\gamma(d)} & \frac{\partial g}{\partial \eta} \\ 0 & L_{\gamma(d)\eta} & L_{\gamma(d)\gamma(d)} & \frac{\partial g}{\partial \gamma(d)} \\ 1 & \frac{\partial g}{\partial \eta} & \frac{\partial g}{\partial \gamma(d)} & 0 \end{vmatrix} / |H^*| = -\frac{\partial g}{\partial \kappa}\left(L_{\eta\eta}L_{\gamma(d)\gamma(d)} - L_{\gamma(d)\eta}L_{\eta\gamma(d)}\right) / |H^*| > 0$$

$$\frac{\partial \eta}{\partial l(d)} = \begin{vmatrix} L_{\kappa\kappa} & 0 & 0 & \frac{\partial g}{\partial \kappa} \\ 0 & 0 & L_{\eta\nu(d)} & \frac{\partial g}{\partial \eta} \\ 0 & 0 & L_{\nu(d)\nu(d)} & \frac{\partial g}{\partial \nu(d)} \\ \frac{\partial g}{\partial \kappa} & 1 & \frac{\partial g}{\partial \nu(d)} & 0 \end{vmatrix} / |H^*| = -L_{\kappa\kappa}\left(L_{\eta\nu(d)} \frac{\partial g}{\partial \nu(d)} - L_{\nu(d)\nu(d)} \frac{\partial g}{\partial \eta} \right) / |H^*| > 0$$

$$\frac{\partial \nu(d)}{\partial l(d)} = \begin{vmatrix} L_{\kappa\kappa} & 0 & 0 & \frac{\partial g}{\partial \kappa} \\ 0 & L_{\eta\eta} & 0 & \frac{\partial g}{\partial \eta} \\ 0 & L_{\nu(d)\eta} & 0 & \frac{\partial g}{\partial \nu(d)} \\ \frac{\partial g}{\partial \kappa} & \frac{\partial g}{\partial \eta} & 1 & 0 \end{vmatrix} / |H^*| = -L_{\kappa\kappa}\left(L_{\eta\eta} \frac{\partial g}{\partial \nu(d)} - L_{\nu(d)\eta} \frac{\partial g}{\partial \eta} \right) / |H^*| > 0$$

Hence the result. ■

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Figure 1a): First period payments

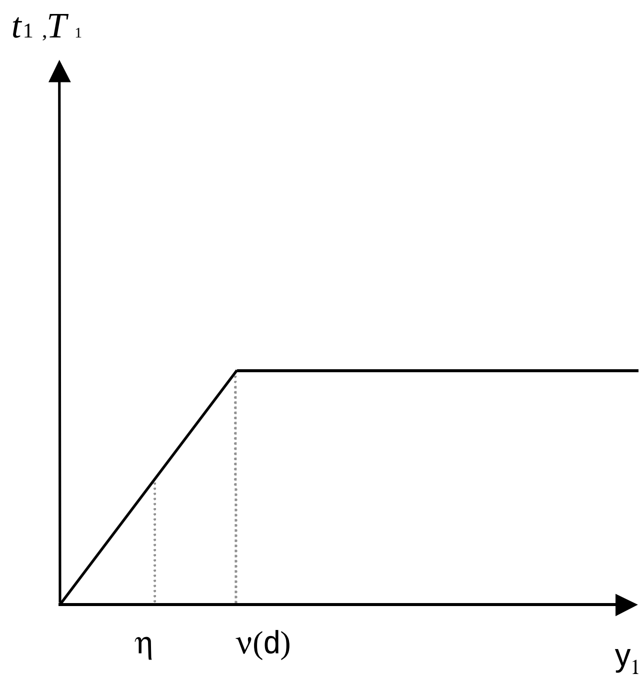

Figure 1b): Second period payments in terms of y_1 (δ_2=0)

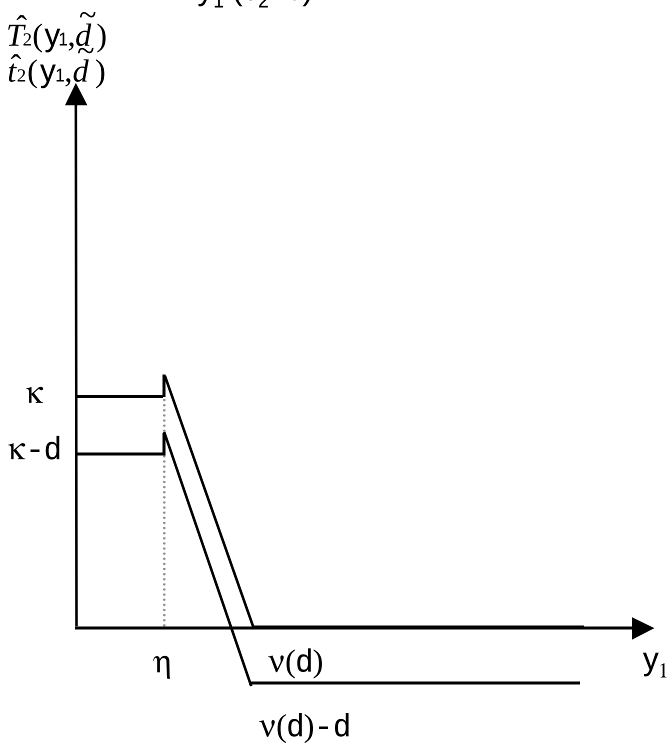

Figure 2a): Second period payments in terms of y_2 (δ_1=1)

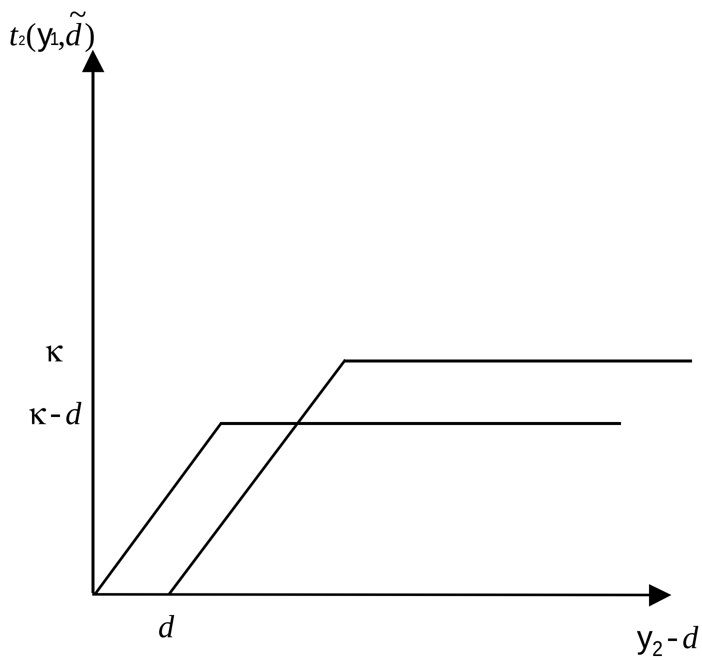

Figure 2b): Second period payments in terms of y_2 (δ_1=0 and $y_1<\nu+pd$)