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Corporate Risk Management: A Model Based on Forward and Volatility Risk Premia

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ABSTRACT

This study proposes an integrated risk management approach for corporations based on the existence of forward and volatility risk premia. The characteristics of those premia are captured by an optimization problem that generates a collection of optimal linear and nonlinear hedging solutions for different risk limits. The proposed approach easily accommodates multivariate exposures and is fully implementable. A theoretical analysis of the sensitivity of the optimal solutions to changes in the forward and volatility risk premia is performed by means of the general implicit function theorem.

Keywords: Risk premia, risk limits, forwards, options and efficient set.
JEL classification: D81 and G30.

RÉSUMÉ

Cette étude propose une approche intégrée de gestion des risques pour les entreprises non-financières, fondée sur l'existence de primes de risques à terme et de volatilité. Un problème d'optimisation est formulé à partir des attributs des primes de risques et offre des solutions de couvertures selon différentes limites de risque. Le problème comporte assez de souplesse pour incorporer une variété d'expositions aux risques et s'implante facilement en pratique. Par le biais du théorème des fonctions implicites, l'étude propose une analyse de sensibilité des solutions optimales aux variations dans les primes de risque.

Mots clés: Primes de risque, limites de risque, taux à terme, options et frontière efficiente.

1 INTRODUCTION

There is a whole range of reasons motivating the management of financial risks. For example, Smith and Stulz (1985) evoke three motives: reduction of the anticipated corporate tax burden, reduction of financial distress costs by lowering the probability of bankruptcy, and, finally, reduction or elimination of financial losses, all of which tend to stabilize the relationship between external financing and the implementation of investment projects. Shimko (1995) contends that corporate risk-hedging practices facilitate long term planning of investment projects, increase debt capacity, and favor the optimal deployment of financial resources[1]. Fenn, Post and Sharpe (1987) assert that top corporate executives take an interest in hedging risks when the firm's profitability affects the size of their compensation. Haushalter (1998) studies the behavior of firms operating in the energy commodities industry. He observes that firms with high financial leverage and limited financial flexibility are more active in risk hedging. Cliche (2000) makes an interesting survey of corporate risk hedging. Her study reveals that hedging is most often a fundamental concern in large corporations operating in unregulated industries. The intensity of hedging activities seems to depend on tax charges, financial distress costs, the presence of restrictive clauses, and on the nature of the firm's investment and financing projects.

Despite the literature exposing the reasons why firms should hedge and sizing up the depths of derivative markets, it seems that firms have only a few pointers as to how an optimal hedging strategy should be implemented. However, some research addressing this issue have recently appeared. Ahn, Boudoukh, Richardson and Whitelaw (1999) go beyond simple hedging schemes and determine the out-moneyness and hedging levels needed to optimize the balance between the option cost and its ability to reduce the Value-at-Risk. In a continuous time setting, Moschini and Lapan (1995) use constant absolute risk aversion and normal distribution to derive optimal production and hedging decisions when futures price, basis and production risks affect the firm. Options become a useful hedging vehicle when the firm's net profit is nonlinearly influenced by the price risk. Brown and Toft (2002) disregard the preference parametrization approach and work out a theoretical model based on the firm's net profit function where optimal hedging strategies based on forwards, options and customized instruments, are proposed under price and quantity risks.

While the forward premium concept plays some role in Moschini and Lapan (1995) and Brown and Toft (2002), our paper specifically focuses on optimal hedging solutions for forwards and plain vanilla options in response to the nature of the forward and volatility premia. Although many versions of the model can be studied, it is assumed that all risk exposures are hedgable and may be hedged only partially given the presence of forward premia. The remaining risk exposures left

[1] In this regard, see also Demarzo and Duffie (1995).

unhedged are covered via options. Thus, the firm eliminates the downside risk associated with extreme events but, in return, is confronted with a volatility risk which influences the configuration of the option choosen. The simplicity of the model proposed herein is such that it can accommodates a significant number of risk exposures and captures the correlations amongst the forward risk premia. Thus, the hedging solutions reflect the diversification benefits observed under different risk exposures of different natures. This is an attribute that market practitioners have recognized since they are confronted with risk exposures of different natures. Additionally, the strong empirical orientation of the approach results in solutions that are fully implementable in practice.

To formalize the understanding with respect to the optimal solutions, the second part of this study is devoted to comparative statics in the spirit of Ahn, Boudoukh, Richardson and Whitelaw (1999). To accomplish this task, the problem is cast into a generalized Lagrangian problem and the Khun-Tucker conditions are examined. The sensitivity of the optimization parameters to changes in the forward and volatility premia is investigated by using the general implicit function theorem. Given that our optimization problem invariably applies a linear or non-linear hedge on every risk exposure, the comparative statics is performed in a distinctive context where a risk limit is imposed as opposed to Ahn, Boudoukh, Richardson and Whitelaw (1999) who parametrize risk through a Value-at-Risk measure.

In the next section, the approach is presented. The third section shows an illustrative example. Comparative statics is performed in the fourth section while concluding remarks are offered in the last section.

2 THEORETICAL APPROACH

Consider a firm that maximizes net economic profit and faces both in time and type, different risk exposures that are all hedgable. The managers of the firm are risk adverse and have an incentive to reduce risk. The firm avoids agency costs by offering to its managers nonlinear compensation contracts tied to the firm's profit. Other considerations such as the structure of long term debt, the capital structure, the portfolio of projects, and the cost structure of the firm have no influence over hedging policies. These elements mainly undergo long term fluctuations, whereas the approach proposed herein focuses on short-term risks.

Empirically driven approaches that focus on hedging often aim at estimating the optimal hedge ratio when risk is modulated by means of forward (or futures) contracts (Bodnar, Hayt and Marston (1996,1998), and Howton and Perfect (1998)). Typically

$$pl_T = (s_T - s_t) - \delta(f_{T,T} - f_{t,T}) \tag{1}$$

where pl_T corresponds to the profit or loss occurring in the period extending from t to T; $s_t(s_T)$ corresponds to the value of the spot variable observed at t (T); $f_{t,T}(f_{T,T})$ corresponds to the value of the related forward contract observed at $t(T)$ and maturing at T; and δ $(0 \leq \delta \leq 1$ with no loss of generality) is the hedging coefficient[2]. When the forward price or rate is an unbiased predictor of the future spot value (absence of forward risk premium, that is $f_{t,T} - E(s_T) = 0$), the risk-averse manager will optimize the value of δ by minimizing the variance of equation (1)[3] since $E(pl_T) = 0$ when the spot rate or price is trendless, $E(s_T) = s_t$ [4]. When the forward price or rate delivers a biased prediction of the future spot value, that is $f_{t,T} - E(s_T) \neq 0$, optimization of the parameter δ must take into account both the mean and variance of equation (1).

Equation (1) can be modified in a number of ways. Many firms base their financial planning on the forward rate structures observed at the beginning of the planning period and use them as reference points for rewarding risk managers in accordance with their ability to reduce hedging costs. Additionally, by generalizing equation (1) to multiple risk exposures and, since by arbitrage $f_{T,T} = s_T$, the yearly profits or losses realized, pl (dropping off its sub-indices t and T) is defined by

$$pl = \sum_{i=1}^{n} \sum_{t=1}^{\tau} N_{it} [\underbrace{(s_{it} - f_{it})}_{pl(nh)} - \underbrace{\delta_i (s_{it} - f_{it})}_{pl(lh)}] \tag{2}$$

where s_{it} is the spot rate associated with the risk exposure of category i observed at t, f_{it}, the corresponding forward rate observed at the beginning of the year and maturing at t; $n(\tau)$ corresponds to the number of risk categories (periods within the year); N_{it} denotes the notional amount associated with the risk exposure of category i observed at period t; and δ_i the linear hedge parameter uniformly applied to all exposures of category i with $0 \leq \delta_i \leq 1$; and where $pl(nh)(pl(lh))$ corresponds to the profits or losses resulting from no hedging (linear hedging) .

When $\delta_i = 1, \forall i$, the manager hedges all exposures entirely with forward contracts at the beginning of the year, so that $pl = 0$. Whenever $\delta_i < 1$, the manager partially covers the exposures of category i and realize a profit or a loss at period t. However, partial hedging based on forward contracts leaves the firm exposed to extreme financial events which can have serious effects on the firm's financial health. To immunize the firm against any significant unfavorable movement, the

[2] Equation (1) illustrates a forward contract hedging strategy. Forward contracts are by far the most popular category of derivatives used in risk hedging (Hentschel and Kothari (2001)).

[3] See Hull (2000), amongst others, for the estimation of δ which minimizes the variance of equation (1).

[4] To reach this goal, the preferred strategy seems linear modelling using unconditional or conditional methodologies. See Ederington (1979), Myers (1991), Kroner and Sultan (1993), Ghosh and Clayton (1996),Lypny and Gagnon (1995), Kavussanos and Nomikos (2000) to name only these few). Others such as Lien and Tse (1998) and Lien and Tse (2000), for example, use the semi-variance in equation (1).

manager can buy options[5].

Define

$$pl(nlh) \quad = \quad \sum_{i=1}^{n}\sum_{t=1}^{\tau}[N_{it}(\alpha_i f_{it} - s_{it})]^{+}(1-\delta_i) - op_{it} \tag{3}$$

as profits and losses stemming from non-linear hedging. $\alpha_i f_{it}$ is the strike of option i expressed as a fraction of f_{it}. Thus, if $\alpha_i = 1$, the related options are at-the-money forward and $op_{it} = |N_{it}|BK$ corresponds to the price of option i, maturing at period t with BK referring to the Black (1976) option price on a forward contract. When $N_{it} < 0$ (> 0), a call (put) is chosen since the firm has a short (long) exposure with respect to the source of risk i. Equation (3) shows that each risk exposure partially covered by the forward contract, has its remaining portion protected by option.

The objective function becomes

$$\max_{\theta} \quad E(pl) = E[pl(nh) + pl(lh) + pl(nlh)] \tag{4}$$

solved for the parameter vector $\theta = (\delta_1, \ldots, \delta_n, \alpha_1, \ldots, \alpha_n)$. The fact that the level of out-moneyness has a non-linear effect on the option premium offers the possibility of identifying an optimal strike level that minimizes the total loss. Also, because equation (4) includes options that hedge the remaining risk exposures, pl displays a left-bounded density function. More specifically, the maximum loss on exposure it according to equation (4) is the option's premium and its level of out-moneyness. Thus, capturing the risk associated with equation (4) with a variance or semi-variance parameter seems inappropriate[6]. It seems natural to presume that the tolerance for these losses depends on the risk aversion and/or on the cyclical profitability of the firm such that it may grant a yearly risk limit to manage the risks. With this view, equation (4) is maximized under a risk limit constraint giving

$$\max_{\theta} \quad E(pl) = E[pl(nh) + pl(lh) + pl(nlh)] \tag{5}$$

$$\text{s.t.}$$

$$L \geq \sum_{i=1}^{n}\sum_{t=1}^{\tau} |N_{it}|BK + (\alpha_i - 1)(1-\delta_i)|N_{it}|f_{it}$$

$$\alpha_i \geq 1, \quad 1 \geq \delta_i \geq 0$$

[5]To exploit the effect of imperfect correlations between sources of risk, the firm could acquire a basket option. Unfortunately, most over-the-counter markets for that type of instrument suffer from a substantial lack of liquidity.

[6]If the remaining portion of the notional exposure were only partially covered with options, the pl density function would no longer be bounded and a value at risk measure could be motivated in this case.

where $L > 0$ denotes the exogenous risk limit set by the firm. By solving problem (5) recurrently for different values of L, a frontier that identifies the best hedging portfolios is obtained [7].

3 AN EXAMPLE

To further understand the hedging problem based on equation (5), an empirical example is briefly presented. To undertake this task, we take the point of view of a Canadian firm that faces risks with respect to its Canadian and US swap portfolios. The firm is exposed to fluctuations in the Libor 3-month, BA 3-month[8] and in the US currency. Two categories of market data are required, the so-called "current" data and the historical data. The current data consist of the relevant implied volatilities and of the forward term structures of the 3-month BA and Libor rates and of the US currency as quoted on Bloomberg on January 14, 2000. The implied volatilities and forward rates are inputs in the Black (1976) model to calculate the price of each option, op_{it}. These data are presented at Table 1 as well as the assumed notional amounts linked to the firm's risk exposures.

The other elements critical to solve equation (5) are the anticipated forward risk premium. Although many approaches are available to reach that goal, we choose to perform an historical simulation where the expected forward premia are estimated by their historical mean counterparts[9]. The historical sample covers the period from June 1982 to January 2000 on a monthly basis. The cash BA and Libor rates extracted from Bloomberg, are observed for maturities of $1, 3, 6$, and 12 months in addition of the 2 year Canadian and US swap rates. After applying standard interpolation and boostrapping methods, the $3, 6, 9$, and 12-month forward rates are derived from the cash rates. The forward rates on the US currency are extracted from the spot term structures of the Canadian and US cash rates through the interest rate parity.

The efficient frontier is derived by recurrently solving equation (5) for the parameter vector $\theta = (\delta_i, \alpha_i)$ for $i = 1, ..., 3$ under different values of L.

The results are presented at Table 2 for a range of risk limits between 0.25 and 6 million dollars. The second column shows the expected profit, $E(pl)$ for each risk limit. A clear compromise arises between these two components. Table 2 also shows the collection of hedging parameters $\delta_i, i = 1, 2, 3$

[7] As mentioned before, the parameters δ_i and α_i uniformly applied to each notional exposure N_{it} irrespective of t. Empirical experiments showed that relaxing that assumption does not produce significant gains and considerably deteriorates the parsimony of the approach.

[8] Banker Acceptance 3-month can be considered the Canadian equivalent of the Libor 3-month rate, and is the reference rate on the floating leg of Canadian interest rate swaps.

[9] The historical simulation is a good choice since it requires no prior parameterization of the processes driving the state variables and preserves the integrity of the multivariate density function of the risk premia. This avoids the delicate problem of estimating trends, variances and correlations governing the combined dynamic behavior of the risk premia. See Duffie and Pan (1997) for an interesting discussion on simulation approaches.

and levels of out-moneyness α_i expressed in million of dollars. For instance, at a risk limit of 3.5 million dollars, the firm expects to save 3.98 million dollars in hedging costs. In this case, $\delta(BA) = \delta(Libor) = 0$ and $\delta(US) = 1$. The optimal solution further indicates to buy individual option to protect against BA 3-month (Libor 3-month) downside risk by setting the corresponding strike level at $\alpha = 1.03$ ($\alpha = 1$). The collection of optimal solutions and the shape of the efficient set displayed at Figure 1 emanate from the characteristics of the forward and volatility premia. As will be shown in the next section, the impact of the latter operates trough the interaction between the implied volatility and the realized or forecasted volatility which drives the option final payoff.

4 COMPARATIVE STATIC

4.1 FIRST ORDER CONDITIONS

The first order conditions with respect to equation (5) are derived and the general version of the implicit function theorem is used to assess the sensitivity of the parameter vector θ at the optimum,θ^*, to changes in the premia. To maintain the analysis tractable, the problem is confined to a univariate framework without suffering from any significant loss of generality since individual options are used as opposed to integrated options that critically depend on correlations. Thus, consider the notation where $f_{it} = f_T$; σ_T^I denotes the implied volatility of an option on a forward contract with a maturity T while σ_T^F denotes the forecasted volatility of the spot asset over the time interval $\{t, T\}$.

To derive the first order conditions when $N = -1$, equation (5) can be written as

$$\begin{aligned} E(pl) &= E[pl(nh) + pl(lh) + pl(nlh)] \\ &= (f_T - E[s_T]) - \delta(f_T - E[s_T]) + (1-\delta)(E[s_T - \alpha f_T]^+ - BKe^{rT}) \\ &= f_T(1-\delta)\left[1 - \frac{E[s_T]}{f_T} + e^{rT}(BK(\frac{E[s_T]}{f_T}, \alpha, \sigma_T^F, r, T) - BK(1, \alpha, \sigma_T^I, r, T))\right] \\ &= f_T(1-\delta)\left[1 - \frac{E[s_T]}{f_T} + e^{rT}(BK_P(\sigma_T^F) - BK(\sigma_T^I))\right] \end{aligned} \quad (6)$$

where r is the appropriate spot interest rate. Under the assumption that the forward contract obeys to a log-normal density function, the option is priced according to the Black (1976) model. In addition, if s_T is log-normal, it can be shown that $E[s_T - \alpha f_T]^+ = BK(\frac{E[s_T]}{f_T}, \alpha, \sigma_T^F, r, T)e^{rt} = BK_P e^{rt}$ [10].

[10] See Hull (2000), amongst others, for a complete proof.

Equation (6) underlines one of the main ideas of this study whereby two types of premia characterize the hedging problem: the forward premium proportional to $E(s_T)/f_T$ and the volatility premium captured by $BK_P(\sigma_T^F) - BK(\sigma_T^I)$. Although it is well documented that $f_T \neq E(s_T)$ for some categories of financial variables, such is not the case when σ_T^I and σ_T^F are compared. Some empirical support for the contention that volatility risk is negatively priced is reported by Pan (2002) and Chernov and Ghysels (2000), amongst others. This suggests that short option position must be compensated with a positive premium and, accordingly, may exhibit a bias in forecasting the future realized volatility. In fact, traders who quote implied volatility must account not only for the volatility forecast, but also for the cost of delta-hedging and the required profit margin Fitzgerald (1998)[11].

In the same spirit, the constraint in equation (5) becomes

$$L \geq f_T(1-\delta)\left(e^{rT}BK + \alpha - 1\right) \tag{7}$$

Given this perspective, the influence of $f_T - E(s_T)$ and $\sigma_T^I - \sigma_T^F$ on the expected profitability, $E(pl)$, is presented at Figure 2 through a numerical example. Assume $T = 1$, $F_T = 3\%$, $\sigma_T^I = 30\%$, $r = 3\%$ and $1\% \leq E(S_T) \leq 4\%$ and $10\% \leq \sigma_T^F \leq 40\%$. Say these numbers identify a firm facing on interest risk exposure. The doted line as well as the area located to the right of it emanate from the specific combinations of forward and volatility premia under which $E(pl) = 0$.

This is justified since quadrant no. 1 regroups situations where the forward risk premium is negative and where the implied volatility, σ_T^I, seems expensive with respect to σ_T^F. In quadrant no. 4, the perceived cheapness of σ_T^I reflected by the negative volatility premium cannot compensate for the negative forward premium which results in the full linear covering of the risk exposures. In contrasts, regions located to the left of the frontier and represented by quadrants no. 2 and 3 show that $E(pl) \geq 0$ primarily because the forward risk premium is positive. The middle area identified by $2a$ ($4a$) implies $E(pl) = 0$ ($E(pl) \geq 0$) since the forward (volatility) premium is insufficiently positive (sufficiently negative) to generate positive expected gains.

The generalized Lagrangian problem

$$\begin{aligned} L(\delta,\alpha) &= (1-\delta)\left[1 - \frac{E[s_T]}{f_T} + e^{r\tau}(BK_P - BK)\right] \\ &- \mu_0\left[(1-\delta)\left[e^{rT}BK + \alpha - 1\right] - L\right] \\ &- \mu_1(1-\alpha) - \mu_2(\delta - 1) - \mu_3(-\delta) \end{aligned}$$

[11] See also Fleming (1999) and James and Colchester (2003).

where $\mu = (\mu_0, \mu_1, \mu_2, \mu_3)$ is the vector of Lagrangian multipliers with $\mu > 0$. The optimal solution, $(\delta^*, \alpha^*, \mu_0^*, \mu_1^*, \mu_2^*, \mu_3^*)$, must satisfy the set of first order conditions, denoted by $c_i, i = 1, ..., 6$,

$$\begin{aligned}
c_1 &= \frac{\partial L(\delta,\alpha)}{\partial \delta} = (1 - \frac{E[s_T]}{f_T}) + e^{r\tau}(BK_P - BK) + \mu_0[e^{r\tau}BK + \alpha^* - 1 - \mu_2 + \mu_3] = 0 \\
c_2 &= \frac{\partial L(\delta,\alpha)}{\partial \alpha} = (1-\delta^*)[N(d_2) - N_P(d_2)] - \mu_0(1-\delta^*)(-N(d_2)+1) + \mu_1 = 0 \\
c_3 &= \mu_0 \left[(1-\delta^*)\left[e^{r\tau}BK + \alpha - 1\right] - L\right] = 0 \\
c_4 &= \mu_1(1-\alpha^*) = 0 \\
c_5 &= \mu_2(\delta^* - 1) = 0 \\
c_6 &= \mu_3(-\delta^*) = 0
\end{aligned}$$

where $N(d_2)$ $(N_P(d_2))$ is the usual parameter found in the Black model under a risk-neutral (physical) probability measure. In addition to these equations, the Kuhn-Tucker conditions imposed on the multipliers μ and on δ and α must be meet otherwise the solutions are not admissible from an optimization perspective. These various conditions are presented in Appendix 1 for $\mu_0 > 0$ and $\mu_0 = 0$. The comparative static analysis focuses on the impact of a change in $f_T - E(s_T)$ and/or $\sigma_T^I - \sigma_T^F$ on the optimal solution $(\delta^*, \alpha^*, \mu_0^*, \mu_1^*, \mu_2^*, \mu_3^*)$. This goal is achieved by means of general version of the implicit function theorem where the following matrix system must be solved

$$\begin{bmatrix}
\frac{\partial c_1}{\partial \delta} & \frac{\partial c_1}{\partial \alpha} & \frac{\partial c_1}{\partial \mu_0} & \frac{\partial c_1}{\partial \mu_1} & \frac{\partial c_1}{\partial \mu_2} & \frac{\partial c_1}{\partial \mu_3} \\
\frac{\partial c_2}{\partial \delta} & \frac{\partial c_2}{\partial \alpha} & \frac{\partial c_2}{\partial \mu_0} & \frac{\partial c_2}{\partial \mu_1} & \frac{\partial c_2}{\partial \mu_2} & \frac{\partial c_2}{\partial \mu_3} \\
\frac{\partial c_3}{\partial \delta} & \frac{\partial c_3}{\partial \alpha} & \frac{\partial c_3}{\partial \mu_0} & \frac{\partial c_3}{\partial \mu_1} & \frac{\partial c_3}{\partial \mu_2} & \frac{\partial c_3}{\partial \mu_3} \\
\frac{\partial c_4}{\partial \delta} & \frac{\partial c_4}{\partial \alpha} & \frac{\partial c_4}{\partial \mu_0} & \frac{\partial c_4}{\partial \mu_1} & \frac{\partial c_4}{\partial \mu_2} & \frac{\partial c_4}{\partial \mu_3} \\
\frac{\partial c_5}{\partial \delta} & \frac{\partial c_5}{\partial \alpha} & \frac{\partial c_5}{\partial \mu_0} & \frac{\partial c_5}{\partial \mu_1} & \frac{\partial c_5}{\partial \mu_2} & \frac{\partial c_5}{\partial \mu_3} \\
\frac{\partial c_6}{\partial \delta} & \frac{\partial c_6}{\partial \alpha} & \frac{\partial c_6}{\partial \mu_0} & \frac{\partial c_6}{\partial \mu_1} & \frac{\partial c_6}{\partial \mu_2} & \frac{\partial c_6}{\partial \mu_3}
\end{bmatrix}
\begin{bmatrix}
\frac{\partial \delta}{\partial Y_j} \\ \frac{\partial \alpha}{\partial Y_j} \\ \frac{\partial \mu_0}{\partial Y_j} \\ \frac{\partial \mu_1}{\partial Y_j} \\ \frac{\partial \mu_2}{\partial Y_j} \\ \frac{\partial \mu_3}{\partial Y_j}
\end{bmatrix}
=
\begin{bmatrix}
-\frac{\partial c_1}{\partial Y_j} \\ -\frac{\partial c_2}{\partial Y_j} \\ -\frac{\partial c_3}{\partial Y_j} \\ -\frac{\partial c_4}{\partial Y_j} \\ -\frac{\partial c_5}{\partial Y_j} \\ -\frac{\partial c_6}{\partial Y_j}
\end{bmatrix}$$

for the vector $[\frac{\partial \delta}{\partial Y_j} \quad \frac{\partial \alpha}{\partial Y_j} \quad \frac{\partial \mu_0}{\partial Y_j} \quad \frac{\partial \mu_1}{\partial Y_j} \quad \frac{\partial \mu_2}{\partial Y_j} \quad \frac{\partial \mu_3}{\partial Y_j}]'$ where $Y = (E[s_T], \sigma_T^F)$ [12]. Although analytical solutions of that system can be provided, the magnitude of the problem refrains from any intuitive explanation. Therefore, once each of the derivatives $[\frac{\partial c_i}{\partial \alpha} \quad \frac{\partial c_i}{\partial \delta} \quad \frac{\partial c_i}{\partial \mu_1} \quad \frac{\partial c_i}{\partial \mu_2} \quad \frac{\partial c_i}{\partial \mu_3} \quad \frac{\partial c_i}{\partial \mu_4}]$, $i = 1, ..., 6$ are analytically obtained[13], the vector $[\frac{\partial \delta}{\partial Y_j} \quad \frac{\partial \alpha}{\partial Y_j} \quad \frac{\partial \mu_0}{\partial Y_j} \quad \frac{\partial \mu_1}{\partial Y_j} \quad \frac{\partial \mu_2}{\partial Y_j} \quad \frac{\partial \mu_3}{\partial Y_j}]'$ is solved numerically.

[12] Y can include other variables such as T and L.

[13] All derivative equations are available upon request.

4.2 NUMERICAL ANALYSIS

To enrich the understanding of problem (5), Table 3 reports the expected profit, the first lagrangian multiplier, μ_0 as well as the value of δ^* and α^* under different risk limits and forward and volatility premia based on the numerical example discussed in the previous section.

Table 3 reveals the apparent insensitivity of the expected profit-risk limit profile to the magnitude of the volatility premium when the forward risk premium offers at least 25 basis points. In these cases, it drives the entire optimization problem irrespective of the forecasted future volatility, σ_T^F. A recurrent pattern occurs whereby at low (high) risk limits, the lagrangian multiplier, μ_0, is consistently higher (lower), the proportion of forward covering, δ^*, is higher (lower) and the option configuration is at-(out-of)-the-money.

The combinations $f_T - E(s_T) \leq 0$ and $\sigma_T^I - \sigma_T^F \geq 0$ do not seem of much interest for the firm, and accordingly, the optimal solutions consist of full linear covering, $\delta^* = 100\%$, implying $E(pl) = 0$. When $f_T - E(s_T) \leq 0$ and $\sigma_T^I - \sigma_T^F \leq -5\%$, the perceived cheapness of the implied volatility triggers a partial linear covering and positive but small $E(pl)$. These situations lack of interest for the firm since the corresponding ratio $E(pl)/\textit{risk limit}$ is very low.

Table 4 presents the findings from the comparative statics. From each scenario emerges five indicators: the Kuhn-Tucker condition described in Appendix 1 under which the optimal solution, $(\delta^*, \alpha^*, \mu_0^*, \mu_1^*, \mu_2^*, \mu_3^*)$, is admissible; the level variations in δ^* and in α^* based on the sensitivity measures $\frac{\partial \delta^*}{\partial x}\Delta x$ ($\frac{\partial \alpha^*}{\partial x}\Delta x$) where $x = (f - E(S_T), \sigma_T^I - \sigma_F^I)$ and on a 10(100) basis point change in the forward (volatility) premium. These metrics are calculated by resorting to the general implicit function theorem for various levels of the forward and volatility risk premia and risk limits.

Consider first the case where $f_T - E(S_T) = 0.50\%$. Irrespective of the magnitude of the volatility premium, at risk limits of 10 and 30 million dollars, the optimal solution is consistent with the Khun-Tucker condition no.5 whereby the risk exposure is partially covered by the forward contract while an at-the-money option provides protection against the downside risk of the remaining exposure. The magnitude of the notional exposure combined with the significant 50 basis point forward premium are such that the risk limit is fully exploited. For larger risk limits such as 50 and 100 million dollars, the optimal solution belongs to Kuhn-Tucker condition no. 2. In this case, the distribution of yearly gains is shifted to the right but exhibits more dispersion because the exposure is left uncovered by the forward contract and the selected option possesses a superior level of out-moneyness accompanied, however, by a lower premium. This phenomenon is robust to the magnitude of $\sigma_T^I - \sigma_T^F$ which supports the view that the forward premium seems the predominent force when it reaches higher levels.

Interestingly,$(\Delta\delta^*, \Delta\alpha^*) = 0$ for shifts in the forward premium when $f_T - E(s_T) \geq 0.50\%$. In other words, for such level of the forward risk premium, δ^* and α^* display optimal values robust to the uncertainty involved in the estimation of $f_T - E(s_T)$ and $\sigma_T^I - \sigma_T^F$. This is comforting for the firm faced with implementation of the hedging positions at the beginning of the planning year. It only requires the manager to formulate a forecast of $f_T - E(s_T)$ that should realize *around* $0.50\% \pm 10$ basis points, to presume on the stability of δ^* and α^*.

The previous diagnostic also applies when $f_T - E(s_T)$ is set at 0.25% while the volatility premium varies from 0 to -10%. Such is not the case, however, when the implied volatility exceeds its forecasted counterpart by 5 or 10%. This is a situation where the manager perceives the implied volatility as being particularly expensive. Therefore, for risk limits in the interval $10 - 50$ million dollars, the optimal solution consists of leaving the exposure partially uncovered by the forward contract while protecting the downside risk via an out-of-the-money option (Kuhn-Tucker condition no.1 in Appendix 1).

Under these scenarios, the optimal solutions $\{\delta^*, \alpha^*\}$ appear significantly sensitive. Consider, for instance, the scenario where $f_T - E(s_T)$, $\sigma_T^I - \sigma_T^F$, and L are respectively 0.25%, 5% and 30 million dollars such that $\Delta\delta^* = 53\%$ and $\Delta\alpha^* = 0.20$ ($\Delta\delta^* = -22\%$ and $\Delta\alpha^* = -0.08$) if $\Delta E(s_T) = 10$ ($\Delta\sigma_T^F = 100$) basis points. Thus, if the expected forward premium deteriotes by 10 basis points, the proportion of the exposure linearly covered increases substantially by 53% from $\alpha = 34\%$ (see Table 3 when $f_T - E(s_T) = 0.25\%$, $\sigma_T^I - \sigma_T^F = 5\%$, and $L = 30$) to $\alpha = 89\%$ while the option becomes out-of-the-money as α^* moves from 1.055 (see Table 3) to 1.255. This important movement in the strike level is motivated by the perception that the volatility is expensive. Contrasting shifts in $\{\delta^*, \alpha^*\}$ occur for a 100 basis point increase in σ_T^F. This can be interpreted as a perceived decline in the relative cost of volatility. This supplementary interest for option hedging translates into the acquisition of more expensive options whose strike coefficient moves from 1.055 to 0.9750 and, consistently, into a reduction of 22% of forward covering. This undesirable situation demands extra forecasting abilities of the forward and volatility premia to confidently implement the positions inspired from $\{\delta^*, \alpha^*\}$.

As expected when $f_T - E(s_T) = 0$, full hedging with $\delta^* = 1$ is often observed. The remaining cases take place when the implied volatility is perceived cheap ($\sigma_T^I < \sigma_T^F$) and involve solutions which avoid linear hedging but protect the risk exposure with an at-the-money option. Further, optimal solutions found for risk limits of 10 and 30 million dollars are robust. With risk limits of 50 million dollars and beyond, the Kuhn-Tucker condition no.10 applies and out-of-the-money options are used when a 10 basis point deterioration in the forward premium reduces α^* by -0.11 and -0.21.

The final situation where $f_T - E(s_T) = -0.25\%$ generally requires the manager to fully cover the risk exposure via forward. Deviations from that rule apply when $\sigma_T^I < \sigma_T^F$ by 10%.

5 CONCLUSION

The purpose of this study was to propose a simple multivariate hedging approach based on forward and option contracts when the forward curves observed at the beginning of the planning horizon provide the benchmark estimates of hedging costs. The approach translates into an optimization problem that generates a collection of linear hedging parameters and strike levels on the options that cover the risk exposures portion left unhedged. The simultaneous consideration of linear and nonlinear hedging instruments constraints the optimization problem to satisfy an exogenous risk limit while traditional volatility measure are useless in this case.

The anticipated forward and volatility premia are the main focus that drive the expected profitability over the planning horizon. To examine the nature of the optimal solutions proposed, comparative statics is performed by of the general implicit function theorem. The analysis reveals that the anticipated forward premium becomes a predominant factor when it is high and generates optimal solutions robust to the uncertainty surrounding the estimation of the anticipated premia. This a fundamental attribute for the firm wishing to implement the approach proposed herein. Contrasting findings are observed when the anticipated forward premium is low or negative and the expected volatility significantly exceeds the implied volatility.

APPENDIX 1

Kuhn-Tucker conditions for $\mu_0 > 0$ (risk limit totally employed) and $\mu_0 = 0$ (risk limit partially employed) under which optimal solutions are admissible.

Case	μ_1	μ_2	μ_3	α	δ		Comment
			$\mu_0 > 0$				
1	$\mu_1 = 0$	$\mu_2 = 0$	$\mu_3 = 0$	$\alpha > 1$	$\delta < 1$	$\delta > 0$	Exposed, option out-of-the-money
2	$\mu_1 = 0$	$\mu_2 = 0$	$\mu_3 > 0$	$\alpha > 1$	$\delta < 1$	$\delta = 0$	All Exposed, opt. out-of-the-money
3	$\mu_1 = 0$	$\mu_2 > 0$	$\mu_3 = 0$	$\alpha > 1$	$\delta = 1$	$\delta > 0$	Impossible
4	$\mu_1 = 0$	$\mu_2 > 0$	$\mu_3 > 0$	$\alpha > 1$	$\delta = 1$	$\delta = 0$	Impossible
5	$\mu_1 > 0$	$\mu_2 = 0$	$\mu_3 = 0$	$\alpha = 1$	$\delta < 1$	$\delta > 0$	Exposed, option at-the-money
6	$\mu_1 > 0$	$\mu_2 = 0$	$\mu_3 > 0$	$\alpha = 1$	$\delta < 1$	$\delta = 0$	All exposed, option at-the-money
7	$\mu_1 > 0$	$\mu_2 > 0$	$\mu_3 = 0$	$\alpha = 1$	$\delta = 1$	$\delta > 0$	Impossible
8	$\mu_1 > 0$	$\mu_2 > 0$	$\mu_3 > 0$	$\alpha = 1$	$\delta = 1$	$\delta = 0$	Impossible
			$\mu_0 = 0$				
9	$\mu_1 = 0$	$\mu_2 = 0$	$\mu_3 = 0$	$\alpha > 1$	$\delta < 1$	$\delta > 0$	Exposed, option out-of-the-money
10	$\mu_1 = 0$	$\mu_2 = 0$	$\mu_3 > 0$	$\alpha > 1$	$\delta < 1$	$\delta = 0$	All exposed, opt. out-of-the-money
11	$\mu_1 = 0$	$\mu_2 > 0$	$\mu_3 = 0$	$\alpha > 1$	$\delta = 1$	$\delta > 0$	Fully hedged
12	$\mu_1 = 0$	$\mu_2 > 0$	$\mu_3 > 0$	$\alpha > 1$	$\delta = 1$	$\delta = 0$	Impossible
13	$\mu_1 > 0$	$\mu_2 = 0$	$\mu_3 = 0$	$\alpha = 1$	$\delta < 1$	$\delta > 0$	Exposed, option at-the-money
14	$\mu_1 > 0$	$\mu_2 = 0$	$\mu_3 > 0$	$\alpha = 1$	$\delta < 1$	$\delta = 0$	All exposed, option at-the-money
15	$\mu_1 > 0$	$\mu_2 > 0$	$\mu_3 = 0$	$\alpha = 1$	$\delta = 1$	$\delta > 0$	Fully hedged
16	$\mu_1 > 0$	$\mu_2 > 0$	$\mu_3 > 0$	$\alpha = 1$	$\delta = 1$	$\delta = 0$	Impossible

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Table 1: The "current" data required for solving equation (5) are presented below. The assumed notional amounts (in million of dollars) linked to the swap portfolio of the firm are cash outflows. The 3-month forward rates for different maturities (3, 6, 9 and 12 months) are extracted from the corresponding spot term structures observed on January 14, 2000. The implied volatilities are extracted from options on futures on BAX for the BA 3-month, on Eurodollar futures for the Libor 3-month and on forward on the US dollar expressed in CAD/USD, maturing in March, June, September and December.

	Spot	3-month	6-month	9-month	12-month
			BA 3-month		
Notional amounts	-	-300	-300	-300	-300
Implied volatilities	-	14.3 %	18.3 %	19.4 %	20.9 %
Term structure	5.2 %	5.6 %	6.0 %	6.3 %	6.5 %
			Libor 3-month		
Notional amounts	-	-400	-400	-400	-400
Implied volatilities	-	7.0 %	10.2 %	12.5 %	14.3 %
Term structure	6.0 %	6.3 %	6.6 %	6.9 %	7.0 %
			US currency		
Notional amounts	-	-5	-5	-5	-5
Implied volatilities	-	6.2 %	5.9 %	5.9 %	5.9 %
Term structure	1.450	1.447	1.444	1.441	1.439

Table 2: The annual expected profit resulting from the optimization problem (5) under various risk limits are presented below. L corresponds to the annual risk limit and $E(pl)$ to the average profit expressed in million of dollars based on the historical simulation. $E(pl(lh))$ and $E(pl(nlh))$ respectively correspond to the average profits and losses based on linear and nonlinear hedging. "Cost" and "Out-moneyness" respectively indicate the sum of all individual option premia and the sum of all levels of out-moneyness expressed in million of dollars. The parameters α_i et δ_i respectively correspond to the optimal level of out-moneyness of option i and the fraction of linear hedging of the risk category i.

						BA 3-month		Libor 3-month		US currency	
L	$E(pl)$	$E(pl(lh))$	$E(pl(nlh))$	Cost	Out-moneyness	α	δ	α	δ	α	δ
0.25	0.33	-3.95	-0.10	0.25	0.00	1.00	76 %	0.00	100 %	0.00	100 %
0.50	0.67	-3.52	-0.19	0.50	0.00	1.00	53 %	0.00	100 %	0.00	100 %
0.75	1.00	-3.09	-0.29	0.75	0.00	1.00	29 %	0.00	100 %	0.00	100 %
1.00	1.34	-2.66	-0.38	1.00	0.00	1.00	5 %	0.00	100 %	0.00	100 %
1.25	1.64	-2.33	-0.41	1.25	0.00	1.00	0 %	1.00	91 %	0.00	100 %
1.50	1.93	-2.03	-0.41	1.50	0.00	1.00	0 %	1.00	79 %	0.00	100 %
1.75	2.23	-1.73	-0.42	1.75	0.00	1.00	0 %	1.00	67 %	0.00	100 %
2.00	2.52	-1.43	-0.43	2.00	0.00	1.00	0 %	1.00	55 %	0.00	100 %
2.25	2.82	-1.13	-0.44	2.25	0.00	1.00	0 %	1.00	43 %	0.00	100 %
2.50	3.11	-0.82	-0.44	2.50	0.00	1.00	0 %	1.00	32 %	0.00	100 %
2.75	3.40	-0.52	-0.45	2.75	0.00	1.00	0 %	1.00	20 %	0.00	100 %
3.00	3.70	-0.22	-0.46	3.00	0.00	1.00	0 %	1.00	8 %	0.00	100 %
3.50	3.98	-0.02	-0.38	2.94	0.56	1.03	0 %	1.00	0 %	0.00	100 %
4.00	4.07	-0.02	-0.28	2.63	1.37	1.05	0 %	1.01	0 %	0.00	100 %
5.00	4.20	-0.02	-0.16	2.19	2.81	1.09	0 %	1.03	0 %	0.00	100 %
6.00	4.28	-0.02	-0.08	1.90	4.10	1.14	0 %	1.04	0 %	0.00	100 %

Figure 1: Efficient set expressed in million of dollars, obtained by recurrently solving equation (5).

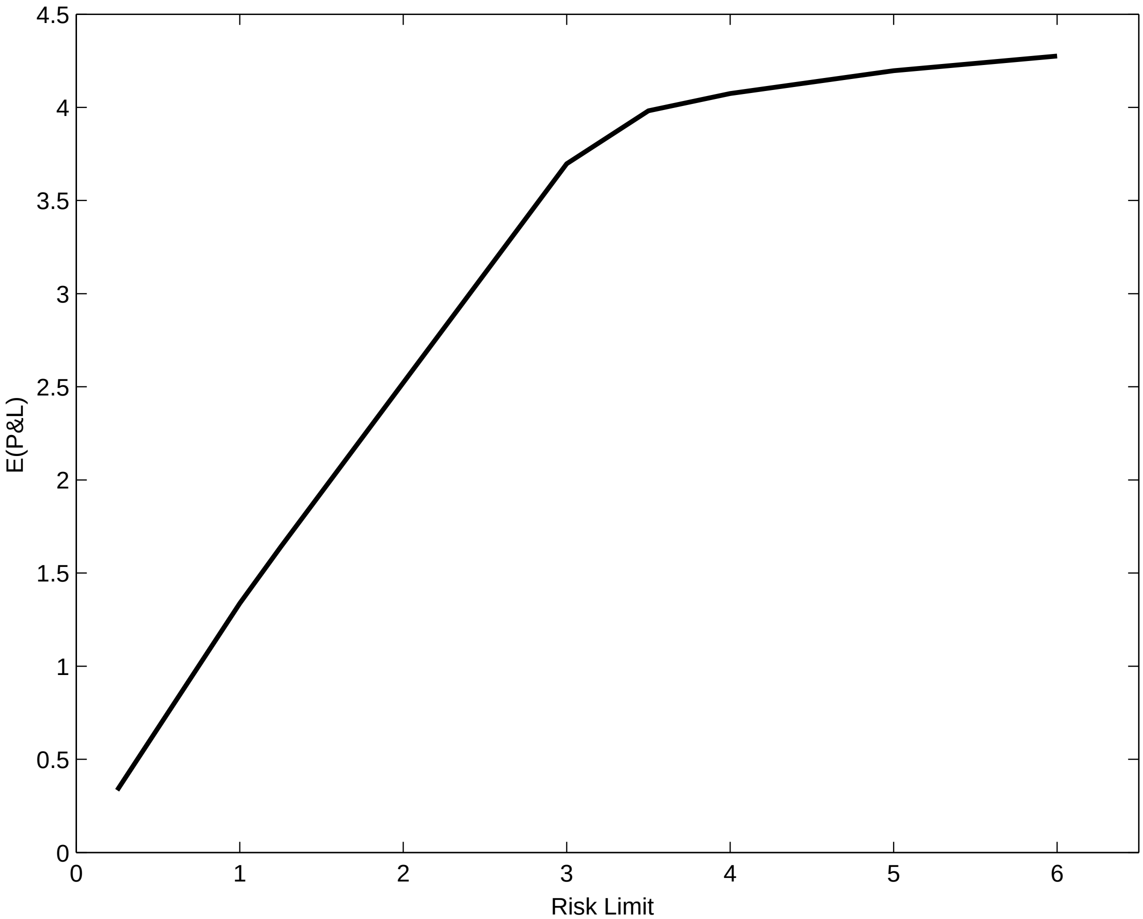

Figure 2: Expected profit under various combinations of the forward and volatility risk premia.

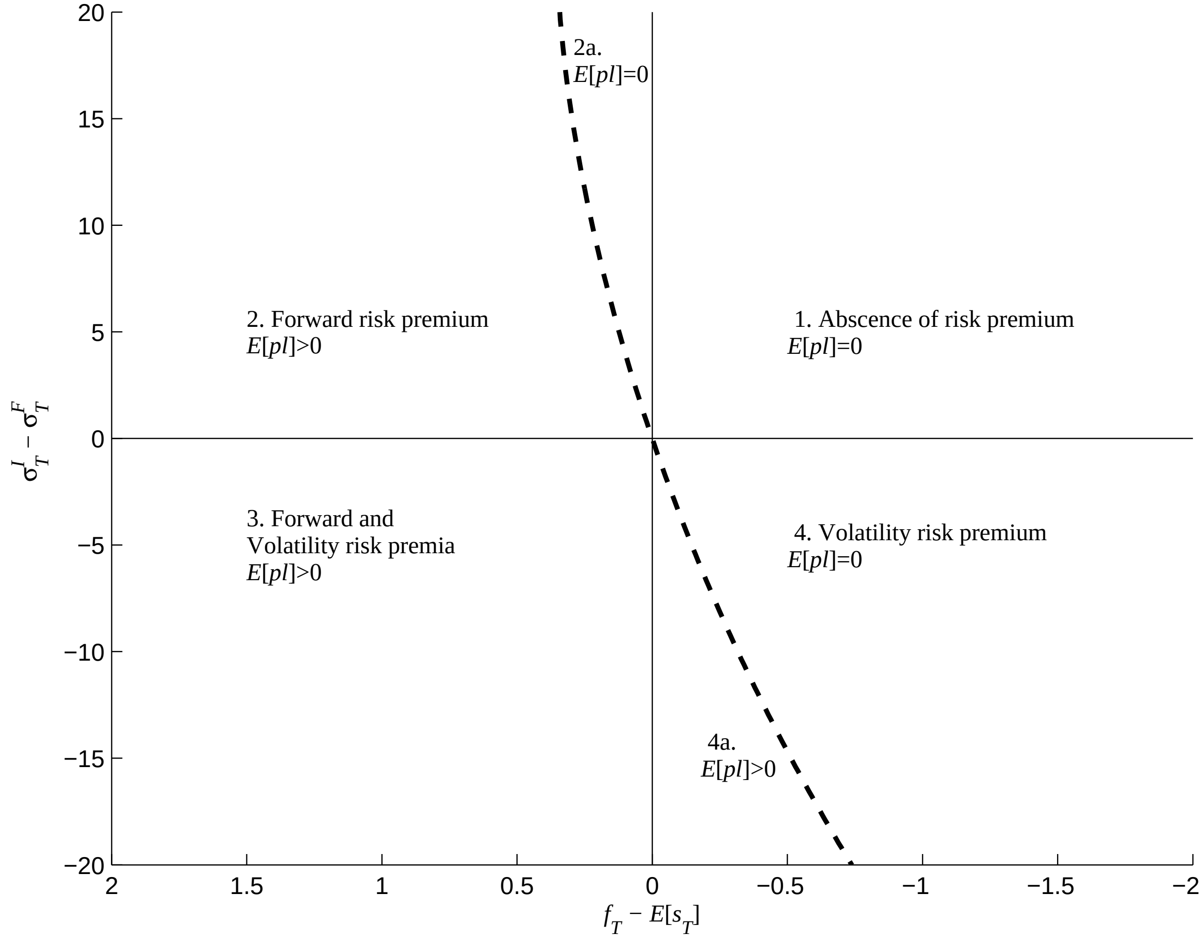

Table 3: Each group of cells reports the expected profit, the Lagrangian multiplier, μ_0, and the optimal δ^* and α^* obtained under different risk limits (in million of dollars) and forward ($f_T - E(s_T)$) and volatility ($\sigma_T^I - \sigma_T^F$) risk premia. The risk exposure notional amount is set to 1000 million dollars, $f_T = 3\%$ and $\sigma_T^I = 30\%$.

		$f_T - E(s_T)$ =0.50%		$f_T - E(s_T)$ =0.25%		$f_T - E(s_T)$ =0.00%		$f_T - E(s_T)$ =-0.25%	
$\sigma_T^I - \sigma_T^F$	L	$E[pl]$,μ_0	δ^*,α^*	$E[pl]$,μ_0	δ^*,α^*	$E[pl]$,μ_0	δ^*,α^*	$E[pl]$,μ_0	δ^*,α^*
10%	10	5, 0.5	0.72, 1.00	2, 0.2	0.90, 1.28	0, 0.0	1.00, 0.00	0, 0.0	1.00, 0.00
	30	16, 0.5	0.16, 1.00	5, 0.2	0.69, 1.28	0, 0.0	1.00, 0.00	0, 0.0	1.00, 0.00
	50	26, 0.4	0.00, 1.08	8, 0.2	0.48, 1.28	0, 0.0	1.00, 0.00	0, 0.0	1.00, 0.00
	100	39, 0.2	0.00, 1.30	15, 0.1	0.00, 1.30	0, 0.0	1.00, 0.00	0, 0.0	1.00, 0.00
5%	10	7, 0.7	0.72, 1.00	2, 0.2	0.78, 1.05	0, 0.0	1.00, 0.00	0, 0.0	1.00, 0.00
	30	20, 0.7	0.16, 1.00	6, 0.2	0.34, 1.05	0, 0.0	1.00, 0.00	0, 0.0	1.00, 0.00
	50	29, 0.3	0.00, 1.08	10, 0.2	0.00, 1.08	0, 0.0	1.00, 0.00	0, 0.0	1.00, 0.00
	100	40, 0.1	0.00, 1.30	17, 0.1	0.00, 1.30	0, 0.0	1.00, 0.00	0, 0.0	1.00, 0.00
0%	10	8, 0.8	0.72, 1.00	3, 0.3	0.72, 1.00	0, 0.0	1.00, 0.00	0, 0.0	1.00, 0.00
	30	23, 0.8	0.16, 1.00	10, 0.3	0.16, 1.00	0, 0.0	1.00, 0.00	0, 0.0	1.00, 0.00
	50	33, 0.3	0.00, 1.08	15, 0.2	0.00, 1.08	0, 0.0	1.00, 0.00	0, 0.0	1.00, 0.00
	100	42, 0.1	0.00, 1.30	20, 0.1	0.00, 1.30	0, 0.0	1.00, 0.00	0, 0.0	1.00, 0.00
−5%	10	9, 0.9	0.72, 1.00	5, 0.5	0.72, 1.00	2, 0.2	0.72, 1.00	0, 0.0	1.00, 0.00
	30	27, 0.9	0.16, 1.00	15, 0.5	0.16, 1.00	5, 0.2	0.16, 1.00	0, 0.0	1.00, 0.00
	50	37, 0.2	0.00, 1.08	20, 0.1	0.00, 1.08	6, 0.0	0.00, 1.05	0, 0.0	1.00, 0.00
	100	44, 0.1	0.00, 1.30	24, 0.0	0.00, 1.30	6, 0.0	0.00, 1.05	0, 0.0	1.00, 0.00
−10%	10	10, 1.0	0.72, 1.00	7, 0.7	0.72, 1.00	3, 0.3	0.72, 1.00	1, 0.1	0.72, 0.00
	30	31, 1.0	0.16, 1.00	20, 0.7	0.16, 1.00	10, 0.3	0.16, 1.00	2, 0.1	0.16, 0.00
	50	41, 0.2	0.00, 1.08	25, 0.1	0.00, 1.08	12, 0.0	0.00, 1.06	2, 0.0	0.00, 0.00
	100	47, 0.1	0.00, 1.30	28, 0.0	0.00, 1.30	12, 0.0	0.00, 1.06	2, 0.0	0.00, 0.00

Table 4: For different values of the forward, $f_T - E(s_T)$, and volatility, $\sigma_T^I - \sigma_T^F$ premia, and of the risk limit L, each group of cells reports the Kuhn-Tucher condition (KTc) presented at Appendix 1 under which there exists an optimal solution, and the level variations in δ^* and α^* for a 10 (100) basis point increase in $E(s_T)$ (σ_T^F). The δ^* (α^*) sensitivity corresponds to $\frac{\partial \delta^*}{\partial x}\Delta x$ ($\frac{\partial \alpha^*}{\partial x}\Delta x$) obtained via the general implicit function theorem and where $x = (f_T - E(s_T), \sigma_T^I - \sigma_F^I)$.

			$f_T - E(s_T) = 0.50\%$			$f_T - E(s_T) = 0.25\%$			$f_T - E(s_T) = 0.00\%$			$f_T - E(s_T)$ =-0.25%	
			$\Delta E(s_T) = 0.1\%$	$\Delta\sigma_T^F = 1\%$		$\Delta E(s_T) = 0.1\%$	$\Delta\sigma_T^F = 1\%$		$\Delta E(s_T) = 0.1\%$	$\Delta\sigma_T^F = 1\%$		$\Delta E(s_T) = 0.1\%$	$\Delta\sigma_T^F = 1\%$
$\sigma_T^I - \sigma_T^F$	L	KTc	$\Delta\delta^*, \Delta\alpha^*$	$\Delta\delta^*, \Delta\alpha^*$	KTc	$\Delta\delta^*, \Delta\alpha^*$	$\Delta\delta^*, \Delta\alpha^*$	KTc	$\Delta\delta^*, \Delta\alpha^*$	$\Delta\delta^*, \Delta\alpha^*$	KTc	$\Delta\delta^*, \Delta\alpha^*$	$\Delta\delta^*, \Delta\alpha^*$
10%	10	**5**	0.00, 0.00	0.00, 0.00	**1**	4%, 0.16	-1%, -0.02	**0**	0.00, 0.00	0.00, 0.00	**0**	0.00, 0.00	0.00, 0.00
	30	**5**	0.00, 0.00	0.00, 0.00	**1**	13%, 0.16	-2%, -0.02	**0**	0.00, 0.00	0.00, 0.00	**0**	0.00, 0.00	0.00, 0.00
	50	**2**	0.00, 0.00	0.00, 0.00	**1**	22%, 0.16	-3%, -0.02	**0**	0.00, 0.00	0.00, 0.00	**0**	0.00, 0.00	0.00, 0.00
	100	**2**	0.00, 0.00	0.00, 0.00	**2**	0.00, 0.00	0.00, 0.00	**0**	0.00, 0.00	0.00, 0.00	**0**	0.00, 0.00	0.00, 0.00
5%	10	**5**	0.00, 0.00	0.00, 0.00	**1**	18%, 0.20	-7%, -0.08	**0**	0.00, 0.00	0.00, 0.00	**0**	0.00, 0.00	0.00, 0.00
	30	**5**	0.00, 0.00	0.00, 0.00	**1**	53%, 0.20	-22%, -0.08	**0**	0.00, 0.00	0.00, 0.00	**0**	0.00, 0.00	0.00, 0.00
	50	**2**	0.00, 0.00	0.00, 0.00	**2**	0.00, 0.00	0.00, 0.00	**0**	0.00, 0.00	0.00, 0.00	**0**	0.00, 0.00	0.00, 0.00
	100	**2**	0.00, 0.00	0.00, 0.00	**2**	0.00, 0.00	0.00, 0.00	**0**	0.00, 0.00	0.00, 0.00	**0**	0.00, 0.00	0.00, 0.00
0.00	10	**5**	0.00, 0.00	0.00, 0.00	**5**	0.00, 0.00	0.00, 0.00	**0**	0.00, 0.00	0.00, 0.00	**0**	0.00, 0.00	0.00, 0.00
	30	**5**	0.00, 0.00	0.00, 0.00	**5**	0.00, 0.00	0.00, 0.00	**0**	0.00, 0.00	0.00, 0.00	**0**	0.00, 0.00	0.00, 0.00
	50	**2**	0.00, 0.00	0.00, 0.00	**2**	0.00, 0.00	0.00, 0.00	**0**	0.00, 0.00	0.00, 0.00	**0**	0.00, 0.00	0.00, 0.00
	100	**2**	0.00, 0.00	0.00, 0.00	**2**	0.00, 0.00	0.00, 0.00	**0**	0.00, 0.00	0.00, 0.00	**0**	0.00, 0.00	0.00, 0.00
-5%	10	**5**	0.00, 0.00	0.00, 0.00	**5**	0.00, 0.00	0.00, 0.00	**5**	0.00, 0.00	0.00, 0.00	**0**	0.00, 0.00	0.00, 0.00
	30	**5**	0.00, 0.00	0.00, 0.00	**5**	0.00, 0.00	0.00, 0.00	**5**	0.00, 0.00	0.00, 0.00	**0**	0.00, 0.00	0.00, 0.00
	50	**2**	0.00, 0.00	0.00, 0.00	**2**	0.00, 0.00	0.00, 0.00	**10**	0.00, -0.21	0.00, 0.00	**0**	0.00, 0.00	0.00, 0.00
	100	**2**	0.00, 0.00	0.00, 0.00	**2**	0.00, 0.00	0.00, 0.00	**10**	0.00, -0.21	0.00, 0.00	**0**	0.00, 0.00	0.00, 0.00
-10.00	10	**5**	0.00, 0.00	0.00, 0.00	**5**	0.00, 0.00	0.00, 0.00	**5**	0.00, 0.00	0.00, 0.00	**5**	0.00, 0.00	0.00, 0.00
	30	**5**	0.00, 0.00	0.00, 0.00	**5**	0.00, 0.00	0.00, 0.00	**5**	0.00, 0.00	0.00, 0.00	**5**	0.00, 0.00	0.00, 0.00
	50	**2**	0.00, 0.00	0.00, 0.00	**2**	0.00, 0.00	0.00, 0.00	**10**	0.00, -0.11	0.00, 0.00	**14**	0.00, 0.00	0.00, 0.00
	100	**2**	0.00, 0.00	0.00, 0.00	**2**	0.00, 0.00	0.00, 0.00	**10**	0.00, -0.11	0.00, 0.00	**14**	0.00, 0.00	0.00, 0.00