Production Flexibility and Hedging

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Abstract

A risk-averse firm faces uncertainty about the spot price of the output, but has access to a futures market. The technology requires both capital and labor to produce the output. Due to the presence of flexibility in production, the level of capital and the volume of futures contracts are chosen under uncertainty (i.e., prior to observing the realized spot price) whereas the level of labor is set under certainty (i.e., after observing the realized spot price). When there is flexibility in production, the optimal production decisions are different between a risk-neutral firm and a risk-averse firm, i.e., the separation result does not hold. Moreover, flexibility in production implies only partial hedging with an actuarially fair futures price, i.e., the full-hedging result does not hold.

Keywords: Hedging, Flexibility, Full-Hedging, Production, Separation.

JEL Classifications: G1, L2.

1 Introduction

There are two central results for the optimal behavior of a risk-averse firm facing a random price and having access to a futures market (Ethier, 1973; Danthine, 1973; Holthausen, 1979; Feder et al., 1980). The first result states that production decisions are unrelated to both the distribution of the random price and risk-aversion. This statement is known as the *separation* result. The second result is called the *full-hedging* result, which states that, under an actuarial fair futures price, the firm hedges by selling the entire production in a futures market regardless of risk aversion. These results do not hold in the presence of multiple sources of uncertainty such as basis risk (Paroush and Wolf, 1992) or production risk (Anderson and Danthine, 1983).[1]

These issues are generally studied in the literature when all production-related decisions of the firm are made under uncertainty, i.e., prior to observing the realization of the random price. In other words, there is no flexibility in production. However, in many industries, the firms are able to adjust production upon acquiring new information about the spot price. While capital inputs require long-term planning, labor inputs can be adjusted more rapidly so as to modify the final level of production. Yet, little is known in the literature regarding optimal behavior when the firms have access to the futures market, but do not have to commit entirely to a certain level of production prior to the realization of the random price. One exception is Moschini and Harvey (1992). They study the effect of flexibility in production on optimal production by comparing the benchmark case of certainty in which the spot price is equal to the futures price with the case of uncertainty in which the spot price is random. They show that in general optimal behavior differ between these two cases.

The purpose of this paper is to study how the presence of flexibility in production affects the separation and full-hedging results. To that end, we consider a technology that requires both capital and labor to produce the

[1] Recently, Dionne and Santugini (2013) showed that, under non-actuarially fair pricing for the futures input market, the separation result does not hold when entry is considered in an imperfectly competitive output market (without production flexibility).

output. The risk-averse firm faces uncertainty about the spot price of the output, but has access to a futures market. Due to the presence of flexibility in production, the level of capital and the volume of futures contracts are chosen under uncertainty (i.e., prior to observing the realized spot price) whereas the level of labor is set under certainty (i.e., after observing the realized spot price).

We present three results. First, we show that in the presence of flexibility in production, the optimal production decisions are different between a risk-neutral firm and a risk-averse firm.[2] Second, we show that the presence of flexibility does not lead to full-hedging under an actuarial fair futures price. In other words, the firm does not hedge expected production because flexibility in production adds a degree of freedom. Third, we consider a specific parametric model with a Cobb-Douglas production function and a symmetric binary distribution. In this parametric case, we show that as long as there is some flexibility in production, the firm hedges partially under an actuarial fair futures price. Hence, hedging and flexibility in production are substitutes. This can explain the behavior of the gold mining industry (Tufano, 1996). In this industry, the firms hedge their selling price for the next three years by using different contracts including forwards and futures. It is well documented they hedge only a fraction of their production (the mean of the industry is 25%) even when their payoff is concave. This means that they keep the flexibility to adjust their production in function of future price fluctuations. In other words, we observe in this industry a trade-off between price protection and production flexibility which rejects full separation. Such trade-off is also observed in the oil and gas industry.

The paper is organized as follows. Section 2 presents the setup. Optimal behavior without and with flexibility in production is provided in Section 3. Finally, Section 4 studies the effect of flexibility in production.

[2]Our result is different from that of Moschini and Harvey (1992). They show that optimal behavior under uncertainty depends on the distribution of the spot price and is different from optimal behavior under certainty when the spot price is equal to the futures price. We consider another aspect of the separation result since we study the effect of risk-aversion on optimal behavior under uncertainty.

2 Preliminaries

Consider a perfectly competitive firm producing a final good using two kinds of input. Specifically, $l \geq 0$ units of labor and $k \geq 0$ units of capital are acquired to produce $q \geq 0$ units of output. The technology to transform the inputs into the output is defined by $q = \varphi(k, l)$ such that $\varphi_1, \varphi_2, \varphi_{12} > 0$ and $\varphi_{11}, \varphi_{22} < 0$. Total cost functions for labor and capital are $c_l(l) \geq 0$ and $c_k(k) \geq 0$, respectively, such that $c_l', c_k', c_l'', c_k'' > 0$.[3]

The firm sells h units of output on the futures market at price F, and sells the remaining $\varphi(k, l) - h$ units on the spot market at price S. Given the firm's decisions $\{k, l, h\}$ and the prices $\{S, F\}$, the profit function is

$$\pi(k, l, h; S, F) = S\varphi(k, l) - c_l(l) - c_k(k) + (F - S)h. \tag{1}$$

The firm is run by a manager who makes decisions so as to maximize the (expected) utility of profit. Specifically, the manager's utility function of profits is $u(\pi(k, l, h; S, F))$ such that $u' > 0$, $u'' \leq 0$.[4] The manager faces uncertainty about the spot price. Let $\tilde{S}$ be the random spot price and $\mathbb{E}[\tilde{S}]$ be the expected spot price where $\mathbb{E}[\cdot]$ is the expectation operator.[5] Assumption 2.1 holds for the remainder of the paper.

Assumption 2.1. *The futures price is actuarially fair, i.e.,* $F = \mathbb{E}\tilde{S}$.

3 Optimal Behavior

Having described the set up, we now study the effect of flexibility in production on the separation result and the full-hedging result. We begin with the definition of flexibility in production. We then recall the optimal behavior for production and hedging when there is no flexibility. We finally derive the optimal behavior of the firm when there is flexibility. In the next section, we

[3]The same analysis can be undertaken with constant unit cost of labor.

[4]Note that risk aversion is not necessary in our analysis, but the payoff function must be concave. Such concavity can be explained by market imperfection such as convex tax functions or asymmetric information in the credit market (Tufano, 1996).

[5]A tilde distinguishes a random variable from a realization.

study the effect of flexibility in production on the separation and full-hedging results.

Definition. Flexibility in production means that the firm is able to alter production once the spot price is observed. Although the degree of flexibility varies across industries, capital-intensive industries (compared to labor-intensive industries) are in general less able to adjust production. For instance, the gold-mining industry require long-term planning in production, which significantly reduces flexibility.

In our model, we assume that capital is chosen prior to observing the spot price whereas labor is chosen after the spot price is known. To fix ideas, consider the Cobb-Douglas production function, i.e., $\varphi(k,l) = k^{1-\alpha}l^{\alpha}, \alpha \in [0,1]$. If $\alpha = 0$, then production exhibits no flexibility since only capital matters. If $\alpha = 1$, then there is full-flexibility in production so that output is essentially set under certainty, i.e., once the spot price is realized.[6] When $\alpha \in (0,1)$, production exhibits a certain level of flexibility, which increases along with an increase in α.

Benchmark Model of No Flexibility in Production. In order to study the effect of flexibility in production, we consider the benchmark case of no flexibility, as usually studied in the literature. To that end, consider the case in which $l = \bar{l} > 0$ is fixed. There is no flexibility in production because output is essentially chosen prior to observing the spot price. In other words, the firm commits to production (via the choice of capital) at the time it chooses the volume of futures contracts. Hence, the firm's maximization problem is

$$\max_{k,h} \mathbb{E}[u(\tilde{S}\varphi(k,\bar{l}) - c_l(\bar{l}) - c_k(k) + (F - \tilde{S})h)]. \tag{2}$$

It follows that the optimal level of capital k^* satisfies

$$F\varphi_1(k^*,\bar{l}) = c_k'(k^*) \tag{3}$$

for both a risk-averse firm (i.e., $u'' < 0$) and a risk-neutral firm (i.e., $u'' = 0$).

[6] When there is full-flexibility in production, the firm faces no risk. Hence, under an actuarially fair futures price the firm has no desire to sell on the futures market.

Moreover, the firm sells all production in the futures market, i.e., there is full hedging,

$$h^* = \varphi(k^*, \bar{l}). \tag{4}$$

Expressions (3) and (4) summarize the separation result and the full-hedging result, respectively, when there is no flexibility in production. See Appendix A for a proof.

General Model with Flexibility in Production. Having recalled the separation and full-hedging results in the absence of flexibility in production, we now state the optimal behavior of the firm when there is flexibility. The maximization problem can be divided into two stages.[7] In the first stage, the firm sets the volume of futures contracts h and acquires the stock of capital k while facing uncertainty about the spot price of the output. In the second stage, given the volume of futures contracts and the capital stock, the firm observes the spot price of the output, and then chooses labor l so that $q = \varphi(k, l)$ units of output are produced. Hence, the firm does not commit to a level of production before uncertainty is resolved, i.e., before the spot price is realized.

We now solve the maximization problem beginning with the second stage. In stage 2, given the firm's decisions $\{k, h\}$ and the spot price S,

$$l^*(k, S) = \arg\max_{l>0} u(S\varphi(k, l) - c_l(l) - c_k(k) + (F - S)h) \tag{5}$$

where all uncertainty has been resolved. The optimal level of labor is implicitly defined by the first-order condition

$$S\varphi_2(k, l) - c_l'(l) = 0 \tag{6}$$

evaluated at $l = l^*(k, S)$. In stage 1, given $l^*(k, S)$

$$\{k^*, h^*\} = \arg\max_{k,h\geq 0} \mathbb{E}u(\tilde{S}\varphi(k, l^*(k, \tilde{S})) - c_l(l^*(k, \tilde{S})) - c_k(k) + (F - \tilde{S})h). \tag{7}$$

[7]See Léautier and Rochet (2012) for a two-stage game in which each firm commits to a hedging strategy in the first stage and then chooses production or pricing strategies in the second stage.

Using the envelope theorem, the first-order conditions are

$$k : \mathbb{E}[(\tilde{S}\varphi_1(k, l^*(k, \tilde{S})) - c_k'(k)) \cdot u'(\Pi^*(k, h, \tilde{S}))] = 0 \tag{8}$$

$$h : \mathbb{E}[(F - \tilde{S}) \cdot u'(\Pi^*(k, h, \tilde{S}))] = 0, \tag{9}$$

$\Pi^*(k, h, \tilde{S}) = \tilde{S}\varphi(k, l^*(k, \tilde{S})) - c_l(l^*(k, \tilde{S})) - c_k(k) + (F - \tilde{S})h$, evaluated at $k = k^*$ and $h = h^*$.

4 Effect of Flexibility in Production

Using (8) and (9), we study the effect of flexibility in production on the separation and full-hedging results. Proposition 4.1 states that in the presence of flexibility in production, the level of capital (and thus the level of output conditional on S) is different between a risk-neutral firm and a risk-averse firm. In general, the production decision depends on risk-aversion.

Proposition 4.1. *In general, flexibility in production removes the separation result, i.e., risk aversion has an effect on the optimal level of capital and thus on the level of production.*

Proof. Consider first a risk-neutral firm, i.e., $u'' = 0$. Then, from (8), k^* is defined by

$$\mathbb{E}[\tilde{S}\varphi_1(k, l^*(k, \tilde{S}))] - c'(k) = 0. \tag{10}$$

Consider next the case of a risk-averse firm, i.e., $u'' < 0$. Suppose to the contrary that k^* for a risk-averse firm is also defined by (10). Then, using (8), it follows that

$$\mathbf{cov}[\tilde{S}\varphi_1(k, l^*(k, \tilde{S})), u'(\Pi^*(k, h, \tilde{S}))] = 0 \tag{11}$$

where $\mathbf{cov}[\cdot, \cdot]$ is the covariance operator. This cannot hold in general since $S\varphi_1(k, l^*(k, S))$ is strictly increasing in S and $u'(\Pi^*(k, h, S))$ is not independent of S. Hence, in general, a risk-averse firm does not behave like a risk-neutral firm. $\square$

Next, Proposition 4.2 states that an actuarially fair futures price does not imply the full-hedging result. Recall from (4) that under no flexibility in production, output is equal to hedging, i.e., $h^* = q^*$. When there is flexibility in production, such statement is not possible since output depends on the observed spot price through the choice of labor. Hence, in that case, following the literature of hedging under exogenous uncertain production (Losq, 1982), the full-hedging result holds when the expected output is equal to the volume of futures contracts. Let $\mu_{q^*} \equiv \mathbb{E}\varphi(k, l^*(k^*, \tilde{S}))$ be the expected optimal level of output.

Proposition 4.2. *Suppose that the firm is risk-averse, i.e., $u''(\pi) < 0$. Then, flexibility in production removes the full-hedging result, i.e., $h^* \neq \mu_{q^*}$.*

Proof. Suppose to the contrary that $h^* = \mu_{q^*}$. Using Assumption 2.1, (9) implies that

$$\mathbf{cov}[\tilde{S}, u'(\Pi^*(k^*, \mu_{q^*}, \tilde{S}))] = 0. \tag{12}$$

This cannot hold in general since $u'(\Pi^*(k^*, \mu_{q^*}), \tilde{S})), S)$ is not independent of S. $\square$

In order to understand further the effect of flexibility on the full-hedging result, we consider the parametric model with a Cobb-Douglas production function, quadratic cost functions, and a symmetric binary distribution for the spot price. Proposition 4.3 compares the optimal level of futures contracts h^* with the expected optimal level of output $\mu_{q^*} \equiv \mathbb{E}\varphi(k, l^*(k^*, \tilde{S}))$. The presence of flexibility (i.e., $\alpha \in (0, 1)$) implies partial hedging when the futures price is actuarially fair. In addition, no flexibility yields the standard full-hedging result whereas full flexibility (i.e., $\alpha = 1$) implies that the firm faces no risk and does not use the futures market.[8]

Proposition 4.3. *Suppose that $\varphi(k, l) = k^{1-\alpha} l^{\alpha}, \alpha \in [0, 1]$, $c_l(l) = wl^2/2$, $c_k(k) = rl^2/2$, and for $\varepsilon \in (0, F)$, $\tilde{S} \sim (1/2 \circ (F - \varepsilon), 1/2 \circ (F + \varepsilon))$. Then, for a risk-averse firm (i.e., i.e., $u''(\pi) < 0$)*

[8]In the case of full-flexibility, output is nonrandom and the distribution of output is degenerate at μ_{q^*}.

1. *For* $\alpha = 0$, $h^* = \mu_{q^*}$.

2. *For* $\alpha \in (0, 1)$, $0 < h^* < \mu_{q^*}$.

3. *For* $\alpha = 1$, $0 = h^* < \mu_{q^*}$.

Proof. See Appendix B. $\square$

A No Flexibility in Production

Since $\varphi_1(k,\bar{l}) > 0$ for all $\bar{l} > 0$, let the inverse function of $q = \varphi(k,\bar{l})$ be $k = \psi(q,\bar{l})$ so that (2) is rewritten as $\max_{q,h} \mathbb{E}[u(\tilde{S}q - c_l(\bar{l}) - c_k(\psi(q,\bar{l})) + (F-\tilde{S})h)]$. The first-order conditions are

$$q : \mathbb{E}\left[(\tilde{S} - c_k'(\psi(q,\bar{l}))\psi_1(q,\bar{l})) \cdot u'(\Gamma(q,h,\bar{l},\tilde{S}))\right] = 0, \tag{13}$$

$$h : \mathbb{E}\left[(F - \tilde{S}) \cdot u'(\Gamma(q,h,\bar{l},\tilde{S}))\right] = 0. \tag{14}$$

where $\Gamma(q,h,\bar{l},\tilde{S}) = \tilde{S}q - c_l(\bar{l}) - c_k(\psi(q,\bar{l})) + (F-\tilde{S})h$. Summing (13) and (14) yields $(F - c_k'(\psi(q,\bar{l}))\psi_1(q,\bar{l}))\mathbb{E}[u'(\Gamma(q,h,\bar{l},\tilde{S}))] = 0$. Since $u' > 0$, it follows that, whether the firm is risk-neutral or risk-averse, the optimal level of output satisfies $F - c'(\psi(q,\bar{l}))\psi_1(q,\bar{l}) = 0$, which is equivalent to (3). Next, let $\mathbf{cov}[\cdot,\cdot]$ be the covariance operator. Given Assumption 2.1, (14) is equivalent to $\mathbf{cov}[\tilde{S}, u'(\Gamma(q,h,\bar{l},\tilde{S}))] = 0$, which is true when $h^* = q^* = \varphi(k^*,\bar{l})$, as stated in (4).

B Cobb-Douglas Production

In stage 2, given $\{k,h,S\}$, the firm's maximization problem is

$$\max_{l>0} u(Sk^{1-\alpha}l^{\alpha} - wl^2/2 - rk^2/2 + (F-S)h). \tag{15}$$

Using (6), the optimal level of labor is

$$l^*(k,S) = \alpha^{\frac{1}{2-\alpha}} w^{-\frac{1}{2-\alpha}} S^{\frac{1}{2-\alpha}} k^{\frac{1-\alpha}{2-\alpha}}. \tag{16}$$

Plugging (16) into $\varphi(k, l^*(k,S)) = k^{1-\alpha}(l^*(k,S))^{\alpha}$ yields the stage-2 optimal level of output as a function of the spot price,

$$\varphi(k, l^*(k,S)) = \alpha^{\frac{\alpha}{2-\alpha}} w^{-\frac{\alpha}{2-\alpha}} k^{\frac{2(1-\alpha)}{2-\alpha}} S^{\frac{\alpha}{2-\alpha}}. \tag{17}$$

Plugging (17) into the profit function yields stage-2 profits as a function of the spot price, i.e.,

$$\begin{aligned}\Pi^*(k,h,S) &= S\cdot\left(\alpha^{\frac{1}{2-\alpha}}w^{-\frac{1}{2-\alpha}}k^{\frac{1-\alpha}{2-\alpha}}S^{\frac{1}{2-\alpha}}\right)^{\alpha}k^{1-\alpha} - w\alpha^{\frac{2}{2-\alpha}}w^{-\frac{2}{2-\alpha}}k^{\frac{2(1-\alpha)}{2-\alpha}}S^{\frac{2}{2-\alpha}} \\ &\quad - rk^2/2 + (F-S)h, \qquad (18) \\ &= (1-\alpha)\alpha^{\frac{\alpha}{2-\alpha}}w^{\frac{-\alpha}{2-\alpha}}S^{\frac{2}{2-\alpha}}k^{\frac{2(1-\alpha)}{2-\alpha}} - rk^2/2 + (F-S)h. \qquad (19)\end{aligned}$$

At stage 1, the firm's maximization problem is

$$\max_{k,h}\mathbb{E}u(\Pi^*(k,h,S)) \tag{20}$$

where $\Pi^*(k,h,S)$ is defined by (19). Using the binary distribution for $\tilde{S}$, k^* and h^* are uniquely defined by the first-order conditions

$$\begin{aligned}k:&\left(\frac{2(1-\alpha)^2\alpha^{\frac{\alpha}{2-\alpha}}w^{\frac{-\alpha}{2-\alpha}}(F-\varepsilon)^{\frac{2}{2-\alpha}}}{2-\alpha}k^{\frac{-\alpha}{2-\alpha}} - rk\right)u'(\Pi^*(k,h,F-\varepsilon)) \\ &= \left(\frac{2(1-\alpha)^2\alpha^{\frac{\alpha}{2-\alpha}}w^{\frac{-\alpha}{2-\alpha}}(F+\varepsilon)^{\frac{2}{2-\alpha}}}{2-\alpha}k^{\frac{-\alpha}{2-\alpha}} - rk\right)u'(\Pi^*(k,h,F+\varepsilon), \qquad (21)\end{aligned}$$

and

$$h: u'(\Pi^*(k,h,F-\varepsilon)) = u'(\Pi^*(k,h,F+\varepsilon)). \tag{22}$$

Since $u'' < 0$, using (19) we solve (22) for h^*, i.e.,

$$h^* = \frac{(1-\alpha)\alpha^{\frac{\alpha}{2-\alpha}}w^{\frac{-\alpha}{2-\alpha}}\left((F+\varepsilon)^{\frac{2}{2-\alpha}} - (F-\varepsilon)^{\frac{2}{2-\alpha}}\right)}{2\varepsilon}(k^*)^{\frac{2(1-\alpha)}{2-\alpha}}, \tag{23}$$

where $k^* > 0$ is defined by the first-order conditions.

Next, let $\mu_{q^*} \equiv \mathbb{E}\varphi(k, l^*(k^*,\tilde{S}))$ be the expected optimal level of output. Using (17),

$$\mu_{q^*} = \frac{\alpha^{\frac{\alpha}{2-\alpha}}w^{-\frac{\alpha}{2-\alpha}}}{2}\left((F+\varepsilon)^{\frac{\alpha}{2-\alpha}} + (F-\varepsilon)^{\frac{\alpha}{2-\alpha}}\right)(k^*)^{\frac{2(1-\alpha)}{2-\alpha}}. \tag{24}$$

Hence, using (23) and (24),

$$\mu_{q^*} - h^* = \left((F+\varepsilon)^{\frac{\alpha}{2-\alpha}} + (F-\varepsilon)^{\frac{\alpha}{2-\alpha}} - \frac{(1-\alpha)\left((F+\varepsilon)^{\frac{2}{2-\alpha}} - (F-\varepsilon)^{\frac{2}{2-\alpha}}\right)}{\varepsilon} \right) \cdot \frac{\alpha^{\frac{\alpha}{2-\alpha}} w^{-\frac{\alpha}{2-\alpha}}}{2} (k^*)^{\frac{2(1-\alpha)}{2-\alpha}}. \tag{25}$$

Finally, it remains to sign expression (25). To that end, let $y = F/\varepsilon$ such that $y \in (0,1)$. From (25), it follows that $\mu_{q^*} - h^* > 0$ if and only if $g(y) > 0$ where, for $y \in (0,1)$,

$$g(y) = (1+y)^{\frac{\alpha}{2-\alpha}} + (1-y)^{\frac{\alpha}{2-\alpha}} - (1-\alpha)\frac{(1+y)^{\frac{2}{2-\alpha}} - (1-y)^{\frac{2}{2-\alpha}}}{y}. \tag{26}$$

To show that $g(y) > 0$, let

$$f(y) = (1+y)^{\frac{2}{2-\alpha}} - (1-y)^{\frac{2}{2-\alpha}} \tag{27}$$

so that

$$f'(y) = \frac{2}{2-\alpha}\left((1+y)^{\frac{\alpha}{2-\alpha}} + (1-y)^{\frac{\alpha}{2-\alpha}}\right) > 0 \tag{28}$$

and

$$f''(y) = \frac{2}{2-\alpha}\frac{\alpha}{2-\alpha}\left((1+y)^{-\frac{2(1-\alpha)}{2-\alpha}} + (1-y)^{-\frac{2(1-\alpha)}{2-\alpha}}\right) > 0. \tag{29}$$

Using the mean-value theorem, and the fact that $f'(y), f''(y) > 0$,

$$\frac{f(y) - f(0)}{y} < f'(y) \tag{30}$$

or

$$\frac{(1+y)^{\frac{2}{2-\alpha}} - (1-y)^{\frac{2}{2-\alpha}}}{y} < \frac{2}{2-\alpha}((1+y)^{\frac{\alpha}{2-\alpha}} + (1-y)^{\frac{\alpha}{2-\alpha}}). \tag{31}$$

Rearranging (31) yields

$$(1+y)^{\frac{\alpha}{2-\alpha}} + (1-y)^{\frac{\alpha}{2-\alpha}} - (1-\alpha/2)\frac{(1+y)^{\frac{2}{2-\alpha}} - (1-y)^{\frac{2}{2-\alpha}}}{y} > 0. \tag{32}$$

Since $\alpha \in (0,1)$, combining (26) and (32) implies that, for $y \in (0,1)$,

$$g(y) > (1+y)^{\frac{\alpha}{2-\alpha}} + (1-y)^{\frac{\alpha}{2-\alpha}} - (1-\alpha/2)\frac{(1+y)^{\frac{2}{2-\alpha}} - (1-y)^{\frac{2}{2-\alpha}}}{y} > 0. \quad (33)$$

Hence, $\mu_{q^*} - h^* > 0$ when $\alpha \in (0,1)$. Using (25), $\mu_{q^*} - h^* = 0$ when $\alpha = 0$. Using (23) and (24), $0 = h^* < \mu_{q^*}$ when $\alpha = 1$.

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