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Corporate insurance with optimal financial contracting★

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Summary. This paper attemps to rationalize the use of insurance covenants in financial contracts, and shows how external financing generates a demand for insurance by risk-neutral entrepreneurs. In our model, the entrepreneur needs external financing for a risky project that can be affected by an accident during its realization. Accident losses and final returns are private information to the firm, but they can be evaluated by two costly auditing technologies. We derive the optimal financial contract: it is a bundle of a standard debt contract and an insurance contract with franchise, trading off bankruptcy costs vs auditing costs. We then analyze how this optimal contract can be achieved by decentralized trading on competitive markets when insurance and credit activities are exogenously separated. With additive risks, the insurance contract involves full coverage above a straight deductible. We interpret this result by showing how our results imply *induced risk aversion* for risk-neutral firms.

Keywords and Phrases: Corporate insurance, Credit, Contract, Audit.

JEL Classification Numbers: D82, G14, G22.

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1 Introduction

Corporate demand for insurance has been regarded for a long time as a consequence of risk aversion at the firm level. Mayers and Smith (1982) were the first to argue that risk aversion provides an unsatisfactory basis for analyzing insurance demand by corporations, since stockholders can eliminate all specific risks in their portfolios through diversification on financial markets. Buying insurance can be interpreted as any financing decision for the firm (or any risk management tool). We know from Modigliani-Miller's theorem that financing only becomes important in the presence of taxes, or contracting costs or when financing policies affect investment decisions.

With risk neutral stockholders, MacMinn (1987) (see also MacMinn and Han, 1990) showed that the existence of transaction costs may induce firms to buy insurance so as to raise the total value of the firm. However, his analysis, while enlightening, is limited on several accounts. First, it assumes that insurance contracts without any transaction costs can be signed. This corresponds to a complete market setting assumption and neglects all informational imperfections that bear on insurance design. Second, the nature of contracts is not endogenously determined; in particular credit contracts are restricted to be debt contracts with an exogenous face value. With these limitations, he shows that insurance (or state contingent claims) may be purchased but he does not really analyze the form of corporate demand for insurance. Third, since both the insurance market and the credit markets involve transaction costs, it remains to understand how these markets interact in shaping the optimal financial and insurance policy.

This paper provides *an optimal contracting framework* within which corporate demand for insurance emerges jointly with financial contracting decisions. In so doing we propose a theory of firm's "induced risk aversion". Moreover, the analysis shows that the exogenous separation of financing and insurance activities[1] entails no social cost despite the externalities between both markets due to the informational link. The need to coordinate the contracts rationalizes the inclusion of compulsory insurance covenants in credit contracts, a widespread practice.

We first analyze the joint credit-insurance contract between a risk neutral entrepreneur and risk neutral financiers, when transaction costs are present on both aspects: namely, both the firm's insurable losses and the firm's cash flows can only be verified through costly auditing. Financial institutions compete by choosing *any type of contract* so as to provide financing and possibly insurance to the firm. We derive the optimal contract in this setting, *without assuming that creditors offer debt contracts*. We *show* that the endogenous optimal contract is a contingent debt contract, where the face value of the debt depends upon accident claims and actual auditing of these claims. This optimal contract can be decomposed as a bundle consisting of a (non-contingent) debt contract and of an insurance contract with franchise.

[1] This separation may be due to historical or legal reasons, or to efficiency gains from increasing returns to specialization.

If insurance companies and creditors face the same transaction costs in assessing risks and controlling for effective losses, the dichotomy in the optimal second-best contract[2] implies that insurance may be provided optimally by insurance companies or by the creditor offering contingent debt payments. If however, as a result of specialization, insurance companies face lower transaction costs in risk assessment, the optimal organization of the market is less obvious. In this context, the main question is whether the separation of insurance markets and credit markets induces inefficiencies, given the strong externality between both markets due to the informational link. We show that the second-best optimum can be implemented by decentralized trading on exogenously separated markets for credit and insurance activities, provided that contracts are verifiable. Creditors then enforce the correct level of coordination between markets either by reacting directly to an existing insurance contract, or by including *compulsory insurance clauses in credit contracts*. Indeed such clauses correspond to the common banking practice of including affirmative insurance covenants in financial contracts (see Zimmerman, 1975). It also provides a rationale for the increase in the coordination of risk management activities observed during the recent years (see Doherty, 1997).

We also show that the second best contract can be decentralized when credit and insurance activities are exogenously separated, even though insurance clauses are not enforceable. Now, the role of the initial credit contract is to create incentives for the firm to buy insurance. This initial contract, however, becomes a debt contract after a renegotiation phase, once the creditor observes the insurance contract signed by the firm.

We devote a special attention to the case of liability insurance, which corresponds to a situation where the final cash flow is the result of two additive independent risks. The insurance contract then involves full coverage above a fixed deductible. It is thus similar to the optimal contract obtained by Townsend (1979) for an individual buying insurance with audit costs. While in Townsend (1979) the demand for insurance results from risk aversion, in our model the firm is risk neutral and the demand for insurance derives solely from the effect of insurance on the face value of the debt. In other words, for a given investment project, a firm buys insurance only to obtain better financial conditions from a creditor that has all the incentives for insurance since it bears all the bad risks under a debt contract. In this context the demand for insurance can be analyzed as the demand of an individual whose preferences are represented by a non-expected utility model, rather than by expected utility maximization (see Machina, 1995). Such preferences are derived by assuming that the firm buys insurance before the credit contract. One can then derive the preferences of the firm for any distribution of net losses. These preferences exhibit a form of risk aversion intermediate between weak and strong risk aversion (see Cohen, 1995).

Our results are related to findings of the recent literature on *background risk* (see Eeckhoudt and Kimball, 1992; Eeckhoudt, Gollier and Schlesinger, 1996;

[2] Second-best efficiency refers to the optimal joint contract; it differs from first-best efficiency to the extent that the optimal joint contract takes into account informational constraints.

Gollier and Pratt, 1996). One interesting result from this literature is that risks may interact under risk aversion even if they are independent. In our paper, we derive the nature of two optimal contracts in the presence of two risks whose realization are private information (Townsend, 1979; Gale and Hellwig, 1985). All agents are risk neutral, but we show that even when the risks are independent, informational imperfections and transaction costs introduce interactions between the two risks and induce risk aversion for a priori risk-neutral agents. As in the literature on background risk under risk aversion, the optimal joint contract differs from the one obtained by considering the two problems independently.

In Section 2, we present our model as well as all assumptions required for the analysis. Section 3 addresses the question of the nature of the optimal contract if financial institutions compete for financing as well as for insurance provision. Section 4 proposes an alternative interpretation of the optimal joint contract as a simple debt contract plus an insurance contract, which allows to conclude that second-best efficiency can be attained even if financing and insurance activities are separated. Section 5 specializes the model to the case of liability insurance. It shows that the optimal contract involves full coverage above a fixed deductible. It also analyzes the demand for insurance using non-expected utility methods. Section 6 focuses on the case where financing and insurance activities are separated and insurance clauses in credit contracts are not enforceable. Section 7 relates our contribution to the existing literature and concludes.

2 The model

A risk-neutral firm needs outside financing for a risky project. It has to borrow I dollars (net of possible internal self-financing) from a potential investor, from a financial institution. The firm is subject to limited liability; its cash-flows, net of any reimbursement, have to be non-negative.

The project involves two successive steps, each one corresponding to a source of risk.

1. The firm has to build some intermediate stock or to buy an asset, which can be damaged by an accident during the course of the project: a random loss l can occur and consequently reduce the future returns of the project. Moreover, the damages cannot be repaired at this stage. The loss from accident is distributed according to a density function $f(.)$ on $[0, L]$, with cumulative distribution $F(.)$.
2. Then, during the second step, the firm uses its accumulated stock or asset to produce and deliver a good to the market. The final cash flow y is uncertain: It depends both upon a non-observable state of nature and upon the first-stage accident loss l, and it is distributed according to a density function $h(., l)$ on $\mathbb{R}_+$, conditional on l, with associated conditional cumulative distribution $H(., l)$.

Banks or any other investor or financial institution that invest funds in a firm do not have perfect knowledge of what is going on in the firm. We follow

Townsend (1979) and Gale and Hellwig (1985) in assuming that, although the firm perfectly observes the damages caused by an accident as well as the realization of the returns of the project, some verification costs have to be spent by the investor in order to have access to such information. Accidents should have a special status in such a setting, since many aspects of an accident, e.g. fire damages, transportation accidents, etc., can be evaluated at a much lower cost than what is involved in the verification of actual cash-flows. Therefore, we assume that the damages l of an accident can be verified by a principal at an auditing cost c_1 whereas final cash flows y can be verified at a cost c_0, which is often interpreted as the cost of bankruptcy,[3] with the assumption that $c_1 < c_0$.

There is perfect competition between investors to finance a given project. Moreover, we start by examining in Section 3 the case of perfect integration of activities, where financial institutions can perform financing as well as insurance activities. In such a case, any investor is supposed to be able to engage in auditing at cost c_1 in the end of period 1 to check the damages the firm pretends to incur, as well as to force bankruptcy at cost c_0 in the end of period 2 so as to get full knowledge of actual cash-flows. We will then focus on the case where, for exogenous reasons, credit and insurance activities are separated: creditors can only perform ex-post auditing of cash-flows, while insurance companies can only use the technology of claim auditing.

All along the paper, we assume the following properties on the distribution of risks:

Assumption A. *(i) For any interior* (y, l), $H_l(y, l) > 0$; *(ii) the inverse hazard rate* $\Psi(y, l) \equiv \frac{1-H(y,l)}{h(y,l)}$ *is decreasing in* y; *(iii) the ratio* $\frac{h(y,l)}{H_l(y,l)}$ *is non-increasing in* y.

A(i) simply assumes that a higher level of accident damages induces lower returns for the project in the sense of first-order stochastic dominance. A(ii) corresponds to the standard Monotone Hazard Rate Property for the conditional distribution of project returns. To interpret A(iii), it is useful to remark that the final returns of the project can be viewed w.l.o.g. as resulting from a non-observable state of nature x at the second step of the project, that is distributed independently of the accident loss l according to the density $g(.)$ with associated cumulative distribution $G(.)$:

$$y = A(x, l),$$

where $A_x > 0$. The cumulative distributions are then related as follows:

$$H(A(x, l), l) = G(x), \tag{1}$$

which implies:

$$hA_l + H_l = 0$$

[3] We must say that we will use this terminology although there are many other aspects to bankruptcy that are not taken into account in such a setting, see e.g. Hart and Moore (1989), Aghion and Bolton (1992), Dewatripont and Tirole (1994).

and $A_l < 0$. A(iii) is then equivalent to the assumption that $A_{lx} \leq 0$. In words, accident losses imply a more severe reduction in final cash-flows when returns are high. As an example, the assumption is satisfied when the returns of the project can be written as: $y = a(l) + xb(l)$, where $b(.)$ is non-increasing. A similar condition is discussed in Jullien, Salanie and Salanie (1998).[4] Let an iso-cumulative curve be a locus along which $H(y,l)$ is constant. Along such a curve, the "marginal rate of substitution" between loss and return is $-\frac{dl}{dy} = \frac{h}{H_l} = -\frac{1}{A_l}$. The assumption means that this rate is decreasing with y.

Note that Assumption A straightforwardly implies that Ψ is decreasing in l :

$$\Psi_l(y,l) \leq 0.$$

We finally assume that the investment decision is optimal given the expected returns of the project and that financing will indeed create non-trivial incentive problems. For that, we assume that:

$$\mathbf{E}[y] \geq I + c_0,$$

which ensures (see the forthcoming analysis) that, in the absence of any auditing technology, financing the project is optimal and requires the elaboration of a standard Gale-Hellwig debt contract that entails bankruptcy with positive probability.

3 Optimal joint contract

Assume that there exists a competitive market for financial contracts, that is, there exists a large number of identical financial institutions, that may write contracts that specify decisions of auditing claims, decisions of imposing bankruptcy and reimbursement plans, contingent on the accident claims and actual reimbursements (or cash-flow reports) by the firm. Perfect competition (or Bertrand competition in contracts) between these companies implies that the equilibrium outcome corresponds to the contract that maximizes the firm's expected payoffs, subject to the constraint that the selected financial institution makes zero-expected profits.

A financial contract under full commitment can be formalized as a direct mechanism. It elicits an accident claim $\hat{l}$ in period 1 and a final cash-flow report $\hat{y}$ in period 2 from the firm, on which several decisions are based. To formalize such a contract let us first consider a stage where a claim l has been audited. The only remaining uncertainty concerns the final cash flow. The continuation contract can be formalized as in Townsend (1979) and Gale and Hellwig (1985). Such a contract will be referred to as $\mathscr{C}[l]$, and is based on two functions agreed upon in the contract:[5] $\mathscr{C}[l] = (\sigma(.;l), T(.,.;l))$ such that

[4] In Jullien et al. (1998), the condition allows to derive well-behaved comparative statics results in risk management problems. It is related to single crossing conditions for comparative statics (see Jewitt, 1989; Athey, 1997).

[5] In models with audit costs, stochastic audit policies "may" be optimal (see Border and Sobel, 1987). In order to keep a well-understood benchmark, we rule out this possibility and assume that

- $\sigma(\hat{y}; l) \in \{0, 1\}$ specifies the probability of verifying the cash-flow after a final report of $\hat{y}$;
- $T(\hat{y}, y; l)$ specifies the amount to be paid by the firm to the financial institution. Of course, this payment depends upon actual final cash-flows y only when it is verified.

If there were no accident, the optimal contract would be a standard debt contract, imposing bankruptcy for $\hat{y} < D$ with $T(\hat{y}, y) = \inf\{y, D\}$. In our context, the overall financial contract can be summarized by an audit function $\tau(.)$ and contingent continuation contracts $\mathscr{C}\left[\hat{l}\right]$ of the type described above:

- $\tau(\hat{l}) \in \{0, 1\}$ specifies the probability of auditing a claim $\hat{l}$;
- $\mathscr{C}\left[\hat{l}\right]$ specifies the continuation contract after a claim $\hat{l}$ is audited and confirmed, or after a non-audited claim $\hat{l}$;
- $\mathscr{C}\left[\hat{l}, l\right]$ specifies the continuation contract when a claim $\hat{l}$ is audited and the true loss l differs from $\hat{l}$.

Such a contract looks quite complicated since it can a priori use accident claims $\hat{l}$ in order to screen the firm according to its true loss. According to the standard argument of the Revelation Principle [see e.g. Myerson (1979) and, in this setting, Border and Sobel (1987)], one can restrict attention to direct truthful mechanisms. As a consequence, it is clearly innocuous to assume extreme punishments for false claims or false cash-flow reports, when these are detected through auditing or bankruptcy: therefore, when an audit invalidates a claim or when bankruptcy invalidates a cash-flow report, all final cash-flows are confiscated by the financial institution. It is thus immediate that $\mathscr{C}\left[\hat{l}, l\right]$ is the contract $\mathscr{C}_0$ that involves full auditing of final cash flow and confiscation of all cash-flows: $\sigma_0(y) = 1$ and $T_0(y) = y$. Similarly all the continuation contracts $\mathscr{C}\left[\hat{l}\right]$ will satisfy $T(\hat{y}, y; \hat{l}) = y$ if $\sigma(\hat{y}, \hat{l}) = 1$ and y differs from $\hat{y}$.

If only revelation constraints with respect to final cash-flow reports were to be considered, the optimal contract would simply be a debt contract contingent on the (assumed truthfully) revealed losses from the accident, characterized by $D(l)$ the face-value of the debt. The reasoning however supposes that the firm honestly reveals its information about accident losses. If the face-value of the debt is smaller for larger accident losses, the firm would indeed not truthfully reveal its losses but would over-report and claim a level of losses equal to L.

Introducing the revelation constraint with respect to accident claims $\hat{l}$ can dramatically affect the nature of the optimal contract. Of course, as noted by Gale and Hellwig (1985), a debt contract is ex-ante incentive efficient, in the sense of Holmström and Myerson (1983); it maximizes the expected utility of the firm under the participation constraint of the investor and the incentive constraints. Since there is no incentive for the firm to misreport an accident at a level inducing auditing, both incentives and efficiency imply that, if a claim is verified and

the financial companies can only commit to a deterministic audit policy. Stochastic audit emerges with risk aversion, while in our model agents are risk neutral.

confirmed, the continuation contract should be a debt contract with a contingent face value $D(l)$.

The main difficulty, however, is that for non audited claims, the contract could try to elicit revelation of l by introducing reimbursement rules, and verification decisions contingent on the report $\hat{l}$. Moreover, the standard reasoning that leads to characterize optimal contracts as debt contracts is not valid anymore. Indeed any attempt to use the continuation contracts to screen between several levels of non-audited accident loss would lead to deviate from a debt contract.[6] A second difficulty is to identify the set of audited accident claims.

Thus optimal contracts could a priori be quite complicated. The following theorem however shows that the optimal contract has a very intuitive structure. When claims are audited, the contract takes the form of a debt contract contingent on the level of observed losses. When claims are not audited, however, truthful revelation of the losses due to the accident imposes a unique debt contract, irrespective of the claim $\hat{l}$: the incentives to exaggerate one's own losses cannot be dealt with by the reimbursement policy so that the contract does not rely at all on alleged losses in this region. Finally, only high claims are audited. In some sense, the theorem proves that sophisticated revelation mechanisms are too costly so that the optimal contract exhibits the simple features of contingent debt contracts with complete pooling of firms with different alleged (and non-audited) accident losses.

Theorem 1. *In case of perfect competition with integrated activities and when* $0 < c_1 < c_0$, *the optimal financial contract is such that there exists* l^* *with* $0 < l^* \leq L$, *such that:*

- *accident claims are not audited within* $[0, l^*]$, *and the contract takes the form of a debt contract with face value* D^*;
- *accident claims are audited within* $]l^*, L]$, *and the contract takes the form of a contingent debt contract with face value* $D_1^*(l)$.
- *For* $l \geq l^*$, $D_1^*(l)$ *is non-increasing, smaller than* D^*, *and the inverse hazard rate is constant at* $\Psi(D_1^*(l); l) = \Psi^*$, *where* $\Psi^* = \mathbf{E}\left[\Psi(D^*, l) \mid l \leq l^*, y = D^*\right]$.

Proof. See Appendix. □

The shape of the optimal contract is pictured in Figure 1. Given the nature of the optimal contract, it needs not be seen as a formal revelation contract. The initial contract is a debt contract with face value D^*. After an accident, the firm may ask for an audit; in case of major damages, the face-value of the debt is then reduced according to the amount of damages when the loss exceeds l^*. Of course, in period 2, the logic is standard: either the firm complies with the contractual payments, or it defaults, in which case it is liquidated by the financial institution.

Let $\mathscr{R}(D, l) \equiv \mathbf{E}\left[\inf\{y, D\} \mid l\right]$ denote the expected payment by the firm under a debt contract, when the loss is l. The participation constraint for the

[6] If only debt contracts are used, non-audited firms will always choose the smallest face value available and no screening can occur.

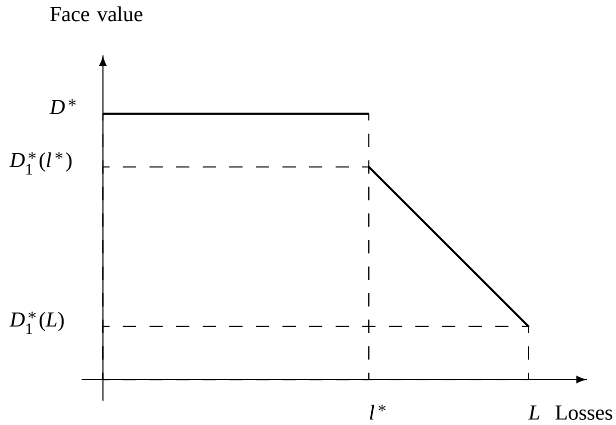

Figure 1. The optimal contract

financial institution can be written:

$$\int_0^{l^*} \mathscr{R}(D^*, l) f(l) dl + \int_{l^*}^{L} \mathscr{R}(D_1^*(l), l) f(l) dl - \int_0^{l^*} c_0 H(D^*, l) f(l) dl \\ - \int_{l^*}^{L} \left[c_0 H(D_1^*(l), l) + c_1\right] f(l) dl \geq I . \tag{2}$$

The expected reimbursement by the firm must cover the loan I plus the transaction costs consisting of the expected bankruptcy cost and the expected audit cost. The basic trade-off behind Theorem 1 can be illustrated using (2). The optimal financial contract relies on a substitution between the two sources of transaction costs. For large damages that would imply large expected bankruptcy costs under a fixed debt contract, the optimal financial contract introduces auditing costs in order to reduce the probability of bankruptcy. More generally the optimal contract minimizes the total audit cost under the participation constraint of the financial institution.

The complete characterization of the optimal contract is not of a particular interest. Yet, by working on the proof of the theorem, one can straightforwardly obtain limit comparative statics results.

Corollary 1. *When $c_1 = 0$, the optimal financial contract involves systematic accident auditing; when c_1 is close to c_0, the optimal financial contract is a fixed debt contract without auditing of accident claims.*

Proof. The result for $c_1 = 0$ is obvious. When c_1 is close to c_0, suppose that $l^* < L$. Increasing l^* by $\epsilon > 0$ implies an increase in the transaction costs given by (2):

$$f(l^*)\left\{c_0 H(D^*, l^*) - c_0 H(D_1^*(l^*), l^*) - c_1\right\}\epsilon \tag{3}$$

which is strictly negative for c_1 in a neighborhood of c_0. As a consequence, the base debt level D^* can be reduced so as to make the investor's participation constraint binding, the reduction being of the order of ϵ can be made small enough so that the revelation constraints still strictly hold. Now, the increase in l^* reduces auditing costs and the decrease in D^* reduces bankruptcy costs; such a profitable change should not be possible at the optimum of the maximization program in the proof of Theorem 1 and therefore, $l^* = L$. □

Let us complete this section by observing that financial contracts are very often renegotiated in real life, in particular after major events that can affect the firm's future profits. It is immediate to notice that the optimal contract described in Theorem 1 is actually robust to renegotiation after the stage where the firm declares its accident damages and is (possibly) audited, and before the realization of y. This comes from the fact mentioned above that a debt contract is ex-ante incentive efficient. After any accident claim and subsequent audit results, it is thus impossible for both parties to find a financial contract that is strictly mutually advantageous, since the best contract would have to be a debt contract with identical expected payments to the investor, and therefore would be identical to the original contract contingent on the announcement or the result of audit.[7]

4 Credit and insurance contracts

Let us now investigate situations where creditors cannot provide insurance and insurance companies cannot provide financing, be it because they are not allowed to for legislative or antitrust reasons, or because they are unable to by lack of experience and expertise in the domain. We still assume that the credit market as well as the insurance market are perfectly competitive. We assume that there is no loading factor in the insurance pricing decisions: perfect competition will then drive insurance companies to charge a fair rate for any insurance policy, where fairness is evaluated with respect to common beliefs about the probability distribution of accident.

We start by proposing an alternative interpretation of the optimal joint contract. Let:

$$r^*(l) \equiv D^* - D_1^*(l).$$

When $l > l^*$, the firm's ex-post revenue is equal to: $\sup\{y - D^* + r^*(l), 0\}$. Therefore, the optimal contract can be seen as a bundle that consists of a standard debt contract with face value D^* and of an insurance contract characterized by

[7] Note that we restrict attention to the possibility of renegotiation after the occurrence of an accident and the conclusion of the audit, and before the realization of final profits; it is well known that renegotiation ex-post, i.e. after realization of final profits, would upset even the standard debt contract, and there may also be scope for renegotiation after a major claim, before actually starting the audit procedure.

a coverage equal to $r^*(l)$, when the loss exceeds the threshold level l^* and no coverage otherwise. The implicit price π^* of the insurance contract corresponds to the fair premium, taking into account auditing costs, which is determined by:

$$\pi^* = \int_{l^*}^{L} (r^*(l) + c_1) f(l) dl \tag{4}$$

We state this interpretation of the optimal joint contract as a corollary, since it underlines the link of our model with the classical finance and insurance literature:

Corollary 2. *The optimal joint contract is a bundle contract consisting of a standard debt contract with face value D^*, and an insurance contract with franchise l^* and coverage $r^*(l)$.*

Proof. Immediate. □

The levels of coverage are computed so as to leave the hazard rate evaluated at the point of bankruptcy unchanged. The insurance part is illustrated in Figure 2.

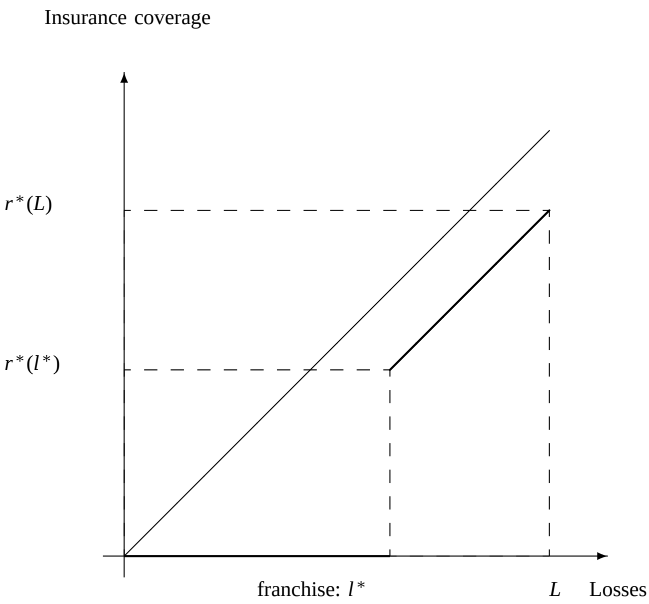

Figure 2. The optimal insurance coverage

Suppose that insurance contracts are verifiable, in the sense that a credit contract could be contingent on the insurance policy acquired by the firm. Corollary 2 then implies that, whatever the timing of transactions, the optimal arrangement with fully integrated activities can be decentralized as the equilibrium outcome in a world of separated activities. First, perfect competition between creditors makes them propose to lend the required investment I plus the necessary insurance premium π^*, under a debt contract with face value D^* that includes a compulsory covenant requiring that the firm acquires an insurance policy for coverage $r^*(l)$, with franchise l^*. Perfect competition among insurance companies makes them willing to offer such insurance policy at the actuarially fair premium π^*. The creditor breaks even since the zero-profit condition (1) can be split into two parts, a zero-profit condition for the credit contract and a zero-profit condition for the insurance contract:

$$\int_0^{l^*} \{\mathscr{R}(D^*, l) - c_0 H(D^*, l)\} f(l) dl + \int_{l^*}^{L} \{\mathscr{R}(D^* - r^*(l), l) + r^*(l) \\ - c_0 H(D^* - r^*(l), l)\} f(l) dl = I + \pi^*.$$

where we use the fact that

$$\mathbf{E}\left[\inf\{y + r, D\} \mid l\right] = \mathscr{R}(D - r, l) + r.$$

Proposition 1. *With separate competition among creditors and insurance companies, the outcome corresponding to the optimal joint contract characterized in Theorem 1 emerges as an equilibrium, when insurance policies are verifiable.*

Proof. Follows from Corollary 2. □

It is indeed part of standard credit practice to incorporate insurance clauses in financial contracts in order to induce the debtor to acquire adequate insurance coverage for his assets. Proposition 1 shows that this practice corresponds to the decentralized market mechanism by which separate transactions with creditors and insurance companies under perfect competition implement the (second-best) efficient outcome corresponding to the optimal joint contract.

The same decomposition as in Proposition 1 can be realized if insurance can be acquired prior to financing. If self-financing resources are sufficiently large, the firm can buy the appropriate insurance policy and then can apply for credit. Observing the adequate insurance policy, competition among creditors will make them propose the optimal debt contract (with total amount borrowed equal to the investment requirements plus the insurance premium net of self-financing resources). Note that, under this timing, insurance policies need not be verifiable but only observable. Such a timing may be reasonable when we consider a firm that already is in business and when insurance is not asset specific.

The nature of the insurance contract is illustrated by looking at the equilibrium probability of bankruptcy. While it is clearly increasing with the loss in the range of non audited claims, we obtain the following result for audited claims:

Corollary 3. *In the range of audited claims, the probability of bankruptcy $H(D^* - r^*(l), l)$ is nonincreasing with the loss l.*

Proof. Omitting the arguments $D^* - r^*(l)$ and l, and using the fact that $\Psi(D^* - r^*(l); l) = \Psi^*$,

$$\frac{d}{dl}H(D^* - r^*(l), l) = H_l - h\frac{\Psi_l}{\Psi_y} = -\frac{h}{\Psi_y}\left(-\Psi_y\frac{H_l}{h} + \Psi_l\right).$$

Using (1),

$$hA_l + H_l = 0,$$

so that

$$\frac{d}{dl}H(D^* - r^*(l), l) = -\frac{h}{\Psi_y}(\Psi_y A_l + \Psi_l).$$

But

$$\Psi(A(x,l), l) = \frac{1 - G(x)}{g(x)}A_x$$

and

$$\Psi_y A_l + \Psi_l = \frac{1 - G(x)}{g(x)}A_{xl} \leq 0$$

which implies the result. □

The insurance contract thus provides partial insurance against the risk of bankruptcy, which can be interpreted as a particular form of co-insurance.

5 Liability insurance and induced risk aversion

To get more specific results, let us specialize the model to the case of liability insurance. In this case, l can be interpreted as a monetary penalty imposed to the firm by some court. The audit cost c_1 can then be interpreted as an administrative cost of verifying the liabilities. Liabilities come in deduction of the cash-flow. Moreover there is no reason for these liabilities to be correlated with other sources of cash-flow. We can thus assume that the final cash flow is additively separable between the accident loss and a random effect:[8]

Assumption LI. *$y = x - l$, where x has a log-concave density on $[\underline{x}, \overline{x}]$, and is independent from l.*

The distribution of x satisfies the monotone likelihood ratio property which ensures that the general assumptions of Section 2 hold and that Theorem 1 is valid. A higher level of accident is thus a ”bad news” as defined in Milgrom (1981). Note that $\Psi(y,l) = \frac{1-G(x)}{g(x)}$ when $y = x - l$, and we let $\psi(x)$ denote this inverse hazard rate. In the case of liability insurance, the distribution of the final cash flow depends only on the net loss $l - r^*(l)$. The optimal contract then takes a particularly simple form.

Proposition 2. *Under the additive separability assumption LI, the optimal insurance contract involves a franchise l^* with full coverage of liabilities above a fixed deductible $d^* < l^*$,i.e. $r^*(l) = l - d^*$.*[9]

[8] Another interpretation is that the asset must be repaid at a monetary cost l.

[9] The distinction we make between ”franchise” and ”deductible” is arbitrary since the insurance literature often considers cases where both coincide (we follow here Townsend, 1979). If the agent is able to costlessly inflate the loss, they coincide in Townsend model (see Picard, 1996). We conjecture that the same holds true in our model.

Proof. The inverse hazard rate is now given by:

$$\Psi(D^* - r^*(l), l) = \frac{1 - G(D^* - r^*(l) + l)}{g(D^* - r^*(l) + l)}$$

Theorem 1 implies that it is constant for $l \geq l^*$, so $l - r^*(l)$ is constant and there exists d^* such that $l - r^*(l) = d^*$. Since $l^* - r^*(l^*) > 0$, d^* is smaller than l^*. □

The remarkable feature of the above insurance contract is that it coincides with the one presented in Townsend (1979). Townsend (1979) was concerned with the optimal insurance policy with costly auditing of the claims for a *risk averse* economic agent. He also characterized the optimal policy as consisting of an auditing of high claims and an insurance coverage with a franchise and a deductible. The close resemblance of our result with Townsend's suggests that the behavior of a risk neutral firm with limited liability, which a priori would yield risk-loving behavior, is deeply affected by the need for outside financing. Outside financing induces the firm to adopt a risk averse behavior.

To make this claim more precise, let us analyze the demand for insurance as follows. Assume that the firm signs the insurance contract prior to the debt contract. The firm then takes into account the impact of insurance on the face value of the debt. This allows to derive the preferences of the firm for any distribution of losses net of insurance reimbursements, and in particular the demand of insurance, for different transaction costs in the insurance market. The induced preferences can be represented by a non-expected utility model that can be analyzed using standard methods (see Machina, 1982, 1987). The fundamental goal of such an approach is to establish a concept of ”induced risk aversion” of the firm, i.e. of risk averse preferences of the pair entrepreneur-investor with respect to insurable risk. This notion seems central to reconcile observed, apparently risk-averse behavior by firms with the standard micro-economic paradigm of the risk neutral firm.

Let E_F be the initial mean of the distribution of loss. An insurance contract will induce a cumulative distribution Φ of losses net of coverage, with mean E_Φ. The corresponding premium is $E_F - E_\Phi + b$, where $E_F - E_\Phi$ corresponds to the actuarial part of the insurance premium while b corresponds to all other costs that insurance companies may support, such as expected verification costs. The firm has to borrow $I + E_F - E_\Phi + b$. For a face value D, the expected reimbursement to the creditor is $\mathbf{E}_\Phi[R(D + l) - l]$, where $R(z) = \mathbf{E}_G[\inf\{x, z\}]$. Facing the final distribution Φ and given the amount to be borrowed, the firm and the creditor sign an optimal (non-contingent) debt contract with face-value $D(\Phi, b)$ that is defined as the smallest value that allows the creditor to meet its minimal profitability constraint, i.e. as the smallest solution of:

$$\mathbf{E}_\Phi\left[R(D + l) - c_0 G(D + l)\right] = I + E_F + b. \tag{5}$$

In all that follows,

– we restrict attention to distributions Φ whose support is included in $[0, L]$,

– we assume that the LHS of (5) is a quasi-concave function of D with maximum larger than the RHS.

The last part of this assumption corresponds to b small enough, which is a necessary condition for insurance to be purchased. It implies not only that $D(\Phi, b)$ exists, but also that it is Fréchet-differentiable with respect to Φ.

Let $V(\Phi, b)$ denote the firm's expected income under the optimal non-contingent debt contract facing a final distribution Φ. $V(\Phi, b)$ defines non-linear preferences on distributions of net losses, that can be used to derive the demand for insurance (see Appendix for details). These preferences can be analyzed through the induced local utility function, $U_{\Phi b}(l)$, that captures the derivative of V with respect to Φ, and defined as follows:[10] for all distributions ϕ,

$$\lim_{\varepsilon \to 0} \frac{V((1-\varepsilon)\Phi + \varepsilon\phi, b) - V(\Phi, b)}{\varepsilon} = \int U_{\Phi b}(l) d\phi(l) - \int U_{\Phi b}(l) d\Phi(l).$$

As discussed in Machina (1987), properties of the local utility function in terms of monotony and concavity translate into local properties of preferences as in the expected utility model.

The analysis of the local utility function allows us to draw conclusions in terms of risk aversion. For non-linear preferences, several concepts of risk aversion have been proposed. Following Cohen (1995), we will say that preferences exhibit *strong risk aversion* if mean preserving spreads are welfare reducing. They exhibit *weak risk aversion* if the mean of a distribution with certainty is always preferred to the distribution itself.[11]

Proposition 3. *Under the additive separability assumption LI, preferences $V(., b)$ exhibit weak risk aversion. Moreover they exhibit strong risk aversion if and only if for all Φ:*

$$\frac{g'(D(\Phi, b) + L)}{g(D(\Phi, b) + L)} + \frac{\mathbf{E}_{\Phi}[g(D(\Phi, b) + l)]}{\mathbf{E}_{\Phi}[1 - G(D(\Phi, b) + l)]} \geq 0.$$

Proof. See Appendix. □

By assumption $g(.)$ is quasi-concave and therefore $g'(.)$ is positive on the lower tail of the distribution. The proposition thus implies that if $D(\Phi, b) + L$ remains in this lower tail, strong risk aversion obtains. Alternatively, notice that the monotone hazard rate property implies that for small accidents losses (L small but not necessarily D), the condition in the proposition is satisfied. The demand for insurance should then have the same general properties than the demand generated by a risk averse expected utility maximizer (see Machina, 1995). This echoes our finding in Corollary 3.

Global concavity of the local utility function may not be satisfied for large accidents. In this case, the preferences will not exhibit strong risk aversion. One can however go one step further, by looking at truncated reduction in risk.

[10] The reader is referred to Machina (1982) for a precise statement in terms of topologies on the sets of functions considered and in terms of a precise definition of Fréchet derivative.

[11] All results in this section are proved in Appendix B.

Corollary 4. *Consider a set W of claims, a coverage function $r(l)$ and an insurance premium $\pi = \mathbf{E}_F[r(l)] + b$. Then there exists d and $r_W(l)$ such that the coverage $r_W(l)$ with premium $\mathbf{E}_F[r_W(l)] + b\}$ is preferred to $r(l)$ with premium π, and $r_W(l) = l - d$ on W and $r_W(l) = r(l)$ outside W.*

Proof. See Appendix. □

Now, suppose that insurance involves a total audit cost that depends on the set $W \subset [0, L]$ of audited claims. An insurance contract specifies an audit for all accident claims in some set W with a coverage $r(l)$, and no coverage outside W ($r(l) = 0$), while b is the cost of auditing W. In this context, $r_W(l)$ corresponds to an insurance contract providing full coverage net of a fixed deductible within W. Thus, *the optimality of full insurance with deductible extends to any audit cost function*. Here again the demand of insurance is similar to the demand of a risk averse agent.

6 Incomplete contracts and renegotiation

When credit and insurance activities are separated, Proposition 1 proves that second-best efficiency can be achieved provided strong requirement can be enforced, such as liquidation of assets if insurance requirement imposed by the credit contract are not exactly fulfilled. This may seem an unrealistic assumption and we consider now a situation where it is not possible to write fully enforceable contracts based on insurance clauses, although insurance policies may be observed by all parties when signed.[12] We restrict ourselves to the situation of additive separability **(**Assumption LI).

The situation is the following. In a first stage, the firm has to get some financing from a creditor which can only audit final cash flows; in a second stage the firm can buy some insurance on the insurance market (specialized in accident auditing), at which point it is bound to one creditor (and one insurance company) under the rules of the contract it has signed (this is a situation of lock in between the firm and the creditor). In a third stage, the lending creditor observes the insurance policy acquired by the firm; the lender can then propose (a take-it or leave-it offer) to renegotiate the credit contract if there is scope for mutual improvement. The question is therefore whether separate markets will achieve second-best efficiency under these working conditions, and what will be the nature of first stage credit contracts in such a setting.

Since the creditor perfectly observes the insurance contract at the renegotiation stage and since it has all the bargaining power, it will reap all the possible renegotiation gains and the firm will end up with the same expected payoff as in the absence of renegotiation. The firm's ultimate payoff will then depend upon both the initial credit contract and the insurance coverage decision and the firm

[12] If the creditor is unable to observe the insurance policy contracted for by the firm, second-best efficiency cannot be attained, since it would require a debt contract under which the firm has no incentives to buy insurance.

will choose the insurance policy that maximizes its expected payoff if the initial credit contract is implemented. The objective of the initial credit contract is therefore to create incentives for the firm to acquire insurance, namely by inducing a concave revenue function for the firm, which implies concave preferences with respect to the net accident loss.

Appendix C provides a formal treatment of the analysis. First, the credit contract allows an initial borrowing of $I + \pi^*$ (so that the firm can afford to buy the optimal insurance policy on top of its investment) and consists of a concave revenue function $\phi(.)$ to the firm. $\phi(.)$ must be chosen so that i) the firm exactly buys the insurance contract (l^*, d^*); ii) the creditor offers to renegotiate a debt contract with face value D^*; iii) after renegotiation the creditor obtains zero profit. As a consequence:

Proposition 4. *Assume assumption LI holds. In an environment with separate competition among creditors and insurance companies, the optimum emerges as an equilibrium when insurance contracts are observable (although not necessarily verifiable) and when creditors can impose any mutually advantageous renegotiation to their debtors.*

Proof. See Appendix. □

There is a lot of freedom in the design of the initial credit contract. Yet, it cannot correspond to a debt contract, which would imply convex revenue for the firm. In the initial contract, control, or systematic verification, is exercised heavily: the creditor is entitled to liquidate the firm and appropriate the verified stream of future profits generated by the firm. The initial entrepreneur is placed as the manager of the firm and is paid in the form of a concave profit participation. Such a contract induces risk averse behavior from the entrepreneur / manager who will thus buy the optimal amount of insurance. Once this insurance decision is made, however, there is no reason to continue implementing the initial contract since it imposes large verification / liquidation costs. The renegotiation may thus be reinterpreted as a leverage buy-out by the manager. As in Hermalin and Katz (1991), the observability of the decision opens the possibility for welfare improving renegotiations and both parties end up signing a (efficient) debt contract, thereby achieving the optimal allocation. In this sense, separate activities between creditors and insurance companies do not impose inefficiencies even in this situation where insurance requirements are not easily enforceable.

7 Link with the literature and conclusion

7.1 Related literature

Our paper relates to the literature on both insurance demand and hedging policy. Following Mayers and Smith (1982), several explanations for insurance demand by corporations have been proposed. As mentioned in the introduction, the paper closest to ours is MacMinn (1987). In a model with costly bankruptcy, he shows that corporations have an incentive to purchase insurance because it may eliminate or reduce bankruptcy costs [see also Smith and Stulz (1985) for a similar argument]. In his model however, bankruptcy costs are born by the firm and debt is taken to be exogenous. Firms buy insurance if expected bankruptcy costs are larger than the loading fee associated with claim costs. As emphasized by Mayers and Smith (1990), this effect will be more significant for small firms since transaction costs are less than proportional to firm size. In our paper we emphasize the effect of insurance on the design of the credit contract (i.e. the face value).

Hedging will be particularly efficient when the marginal cost of external funds increases with the amount raised or when external sources of financing become more costly than internally generated funds. Like any hedging activity, insurance can help corporations maintain sufficient internal funds in loss states to take advantage of investment opportunities. Following Froot, Scharfstein and Stein (1993), insurance can then be viewed as an instrument for coordinating corporate investments and financing policies: it is used as hedging against future transaction costs. In our model, insurance aims at reducing current transaction costs.

A different line of research starts from issues related to managerial incentives. Hedging is desired by managers-stockholders who are not fully diversified (see Stulz, 1985; DeMarzo and Duffie, 1991). The lack of diversification may be the result of an agency problem between shareholders and managers, in which case insurance can reduce agency costs, as in Campbell and Kracaw (1987). Mayers and Smith (1982) suggest that insurance firms may have advantage over outside stockholders in monitoring certain aspects of real activities (such as prevention). DeMarzo and Duffie (1995) analyze risk management in a context where risk averse managers are motivated by future career opportunities. They argue that, due to accounting procedures, hedging modifies the information revealed to the market on managers' abilities. In our model, stockholders have a passive role and agency problems are confined to the relationship between the firm and the financial intermediaries.

Another strong motivation for corporate insurance purchase lies in the nature of tax laws (Main, 1983; Mayers and Smith, 1982; Smith and Stulz, 1985). For example, the presence of a particular tax code may introduce a convex tax function for low levels of taxable income and a linear one for higher income. The convexity implies that corporations have expected tax liabilities greater than the tax liabilities associated with their expected pre-tax income under insurance.

Mayers and Smith (1982) also suggested that insurance firms have a comparative advantage in processing and administrating claims and loss-prevention projects which favors insurance purchasing [see also Doherty (1997) for a discussion]. In this paper we have considered some of these issues but in a different perspective. The emphasis has been put on the nature of the different contracts instead of that of the different organizations.

Another motive for insurance is that it may increase incentives for investments: stockholders of a firm with outstanding risky bonds may have incentives to reject positive net present value projects if the benefits of the project accrues to the bondholders. MacMinn (1987) shows how a bond requiring insurance can reduce such agency problems by reducing the costs of financial distress (see also MacMinn and Han, 1990). These motivations for corporate insurance have not be considered in this research.

More recently, Grace and Rebello (1993) and Rebello (1995) have studied the insurance demand for a firm with private information on expected cash flows and expected insurable losses. They assume that firms with high operating revenues also face high insurable risks. One role for insurance is therefore to signal the quality of the firm to the financial markets. In their framework, favorable information is signaled by the purchase of high insurance coverage. Rebello (1995) also shows how insurance contracting affects capital structure decisions. For example, a firm that chooses full insurance will prefer debt financing while a firm with self-insurance will be indifferent between debt and equity. As indicated above, their information problems are ex-ante while those studied in this article are ex-post. Another important difference lies in the timing of the decisions. In their models, insurance and financing decisions are simultaneous. They claim that identical results would be obtained with sequential decisions without any further restriction if insurance decisions are made first while a commitment on insurance at the time of financial decisions is necessary to get the same results when financial decisions are made first.

In a useful analysis of the competition between insurance and capital markets in the provision of risk management tools, Doherty (1997) distinguishes two types of risk management strategies: hedging (or risk reduction) and cost reduction of risk. Usually hedging activities are risk specific while cost reducing activities are not. Insurance is the hedging instrument for accident or insurable risk but its volume can be reduced if other cost reducing activities become more efficient. Tax linearization is an example, changing leverage to obtain lower expected costs of bankruptcy is another one. Inefficiencies in the insurance markets can also reduce the relative volume of insurance. It is then important that imperfections in different markets be treated symmetrically to obtain adequate predictions about different markets. Our analysis goes precisely in that direction.

7.2 Concluding remarks

In a situation where a firm faces two types of risk in the development of a new project, insurable accident risks and risks on final returns and cash-flows, we have analyzed optimal contracting relationships of this firm with financial institutions, with or without separation of credit and insurance activities. Optimal contracting indeed trades off between the costs of imposing auditing of the final cash-flows of the firm and the costs of auditing alleged accident losses that can be insured. The nature of the optimal contract proves to be a simple bundle between a debt contract and an insurance package. Moreover, the exogenously imposed separation of credit and insurance activities does not cause major inefficiencies in our model. In order to make a case against separation, one should know extremely fine details about renegotiation processes. These negotiation processes could be linked with the degree of concentration in the capital markets, a point that our paper does not address.

Introducing imperfect competition in our model is, however, possible. Indeed, keeping perfect competition in the insurance market when separate, it is easy to see that when there exists a monopoly in the credit sector, the same type of analysis holds true because maximizing the creditor's objectives (either with a joint contract or with a simple financing contract) amounts to a shift in the individual rationality positions, which has the same effect as changing the value of investment expenditures I. Our remarks about separation seem therefore robust to the possibility of market power in the credit sector. It is however the scope of future research to investigate the case where the insurance sector also exhibits market concentration.

We have assumed that after an accident, the destroyed asset could not be replaced. The most natural extension will be to endogeneize the initial investment and to allow for rebuilding of the stock at some replacement costs. The contract should then involve a cash payment at the replacement stage, and the firm will have the choice between investing to replace the asset or a financial investment. This introduces a new agency problem because at this stage the firm is under a debt contract which distorts its investment decisions.

A Proof of Theorem 1

As a preliminary remark, note that, similarly to the Gale-Hellwig model, existence of an optimal contract is not an issue here: as shown below, any contract payment can be implemented through a contingent debt contract, and the set of admissible levels of debt can be a priori bounded. Finally, the set of constraints is not empty since a simple debt contract can ensure financing, which is profitable by assumption, even without insurance.

Now, a direct proof of the result turns out to be extremely problematic. Instead, we take the following route. We first assume that imposing bankruptcy allows the financial institution to perfectly observe both y and l. We derive the optimal contract in this case and show that it does not rely on this assumption about separate observability of y and l. We conclude that this contract corresponds to the optimal contract in our setting, since it remains feasible with observability of y under bankruptcy and since it satisfies truthful revelation constraints with respect to accident damages.

So, let us assume that bankruptcy allows the financial institution to observe (l, y). First it is clear that if an accident is audited and the claim is found to be false, maximal punishment implies that the continuation contract is $\mathscr{C}_0$. A continuation contract following an accident claim that is audited and confirmed takes the form of a financial contract $\mathscr{C}[l]$, with $T(\hat{y}, y, l) = y$ if $\sigma(\hat{y}, l) = 1$ and y differs from $\hat{y}$. For a non-audited claim, the contract specifies a transfer $T(\hat{y}, y, \hat{l}, l)$ if the cash flow is audited. Lying with respect to insurance claim can be deterred when bankruptcy occurs in period 2 by imposing maximal punishment for the firm. Therefore : $\sigma(\hat{y}, \hat{l}) = 1 \Longrightarrow T(\hat{y}, y, \hat{l}, l) = y$, if $\hat{y} \neq y$ or if $\hat{l} \neq l$.

Let us first simplify the notation. The contract is summarized by:

- $\tau(\hat{l})$, the probability of auditing the claim $\hat{l}$,
- $\sigma(\hat{y}, \hat{l})$, the probability of verification of final cash-flows $\hat{y}$ for a claim $\hat{l}$ that is audited and confirmed or that is not audited,
- $D(\hat{l})$, the payment after claim $\hat{l}$ if cash flow is not audited,

and $d(y, l)$, the payment after claim $\hat{l}$ if bankruptcy occurs and claims are confirmed.

The incentives constraints relative to final cash-flow disclosure are standard; they require that:

$$y < D(l) \Longrightarrow \sigma(y, l) = 1$$

and they impose the following condition on payments:

$$\sigma(y, l)\left[\inf\{D(l), y\} - d(y, l)\right] \geq 0$$

Given the expected payment for the firm contingent on an announcement l, there exists a debt contract that yields the same payment; it is determined by a face value $\Delta(l)$ that is the unique (by the monotone hazard rate property) solution of:

$$\mathscr{R}(\Delta(l), l) = \int_{\mathbb{R}_+} \{\sigma(y, l)d(y, l) + (1 - \sigma(y, l))D(l)\}\, h(y, l)dy$$

If $\Delta(l) > D(l)$, a debt contract with face value $\Delta(l)$ always implies ex-post payments that are equal to $\inf\{y, \Delta(l)\}$ and therefore larger than $\inf\{y, D(l)\}$. The equality that defines $\Delta(l)$ above can then be satisfied only for $\Delta(l) \leq D(l)$. Now, in the debt contract with face value $\Delta(l)$, the probability of audit equals: $\int_0^{\Delta(l)} h(y, l)dy$, which is therefore smaller than the probability of audit under the original contract for l, provided the original contract was not itself a debt contract. As a consequence, replacing the original contract for all l by debt contracts with face value $\Delta(l)$ would not affect the firm's payments *under truthful revelation*, but it would reduce the overall probability of audit if there is a positive measure set of l for which this modification could be made, thereby improving the firm's objectives (and relaxing the investor's participation constraint). We now show that if the initial contract is not a contingent debt contract, the contingent debt contract obtained by using face values $\Delta(l)$ improves welfare, even when incentive constraints are taken into account.

Consider the truthful revelation constraints with respect to accident claims. Let $\mathscr{L}_0$ denote the set of claims that are not audited, i.e. $\{l \in [0, L]; \tau(l) = 0\}$, and $\mathscr{L}_1 = [0, L]/\mathscr{L}_0$, its complementary set within $[0, L]$. Note first that the firm would not lie by pretending $\hat{l}$ with $\hat{l} \in \mathscr{L}_1$, since this lie would be discovered, and would trigger maximal payments and zero final profits. As a consequence contracts contingent on $\hat{l} \in \mathscr{L}_1$ are debt contracts. When the firm lies by pretending $\hat{l} \in \mathscr{L}_0$, it also has the option of lying about its final cash-flows. If the firm announces $\hat{y}$ such that $\sigma(\hat{y}, \hat{l}) = 1$, then the lie about $\hat{l}$ will be discovered and the firm will transfer all its final cash-flows y; if $\sigma(\hat{y}, \hat{l}) = 0$, the firm will pay $D(\hat{l})$. Therefore, the firm's deviation will make it pay the minimum of y and $D(\hat{l})$, i.e. what is paid under a standard debt contract with face value $D(\hat{l})$ (provided there exists a level $\hat{y}$ that avoids bankruptcy). For the optimal contract, let D_0 denote the minimum of $D(\hat{l})$ within $\mathscr{L}_0$. At the optimum, the truthful revelation constraints can therefore be summarized by: $\mathscr{R}(\Delta(l), l) \leq \mathscr{R}(D_0, l)$ i.e. $\Delta(l) \leq D_0$, for (almost) all l. D_0 is finite since otherwise the revelation constraint is nowhere binding.

Suppose that for some $l \in \mathscr{L}_0, D_0 < D(l)$. For a positive mass of y: $\sigma(y, l) = 1$ and $d(y, l) < \inf\{y, D(l)\}$. One can then increase $d(y, l)$ for these realizations and reduce both $D(l)$ and $\mathscr{L}_1$ without changing the expected payment. This would reduce the expected auditing costs without affecting incentive constraints, which cannot be possible at the optimum. Therefore $D(l) = D_0$ for $l \in \mathscr{L}_0$ and $D(l) = \Delta(l) \leq D_0$ for $l \in \mathscr{L}_1$.

Suppose now that for a set of positive mass $\mathscr{K} \subset \mathscr{L}_0,\ \Delta(l) < D_0$. Consider the following manipulation. First replace the contract for all $l \in \mathscr{K}$ by a debt contract with face value D_0. This increases total expected payment and strictly reduces auditing costs since in any case cash-flows $y < D(l) = D_0$ must be audited. Then reduce uniformly D_0 by δ on $\mathscr{L}_0$ so as to restore total expected payment at its initial level. In the process stop auditing cash-flows $l \in \mathscr{L}_1$ such that $\Delta(l) > D_0 - \delta$. This again reduces (weakly) auditing costs. Overall expected payment is unchanged and auditing costs are reduced, which contradicts the optimality of the contract. As a consequence $\Delta(l) = D_0 = D(l)$ for all claims not audited.

The optimal contract is therefore a contingent debt contract, with a face value $D_0 = D^*$ when no audit occurs and $D(l) = D_1^*(l)$ with an audit of l. It can be completely characterized by the following program:

$$\max_{D_0, D(l), \tau(l)} \quad -\int_0^L \{c_0 [\tau(l)H(D(l), l) + (1 - \tau(l))H(D_0, l)] + c_1\tau(l)\} f(l)dl$$

$$\text{s.t.} \quad \tau(l) \in \{0, 1\} \text{ and } 0 \leq D(l) \leq D_0$$

$$\int_0^L \Big\{\tau(l) \left[\mathscr{R}(D(l), l) - c_0H(D(l), l) - c_1\right] + (1 - \tau(l)) \left[\mathscr{R}(D_0, l) - c_0H(D_0, l)\right] \Big\} f(l)dl \geq I$$

where the constraint ensures that the financial institution meets its participation constraint. Let ρ denote the Lagrange multiplier associated to this participation constraint. After straightforward manipulations, the FOC can be written:

$$\tau^*(l) = 1 \Rightarrow D^* > D_1^*(l) > 0 \text{ and } \Psi\left(D_1^*(l), l\right) = \Psi^* \equiv \frac{1+\rho}{\rho}c_0 \tag{6}$$

which defines $D_1^*(l)$, decreasing,

$$\int_{\mathscr{L}_0} \left[\Psi\left(D^*, l\right) - \Psi^*\right] h(D^*, l) f(l) dl = 0. \tag{7}$$

Optimality of the audit policy requires that if $\tau^*(l) = 1$:

$$\mathscr{R}(D_1^*(l), l) - \mathscr{R}(D^*, l) - \Psi^* \left[H(D_1^*(l), l) - H(D^*, l)\right] \geq \frac{1+\rho}{\rho} c_1 \tag{8}$$

Let us fix $D_1^*(l)$ and D^*. When $D_1^*(l) > D^*$, truthful revelation conditions reduce to $\tau^*(l) = 0$. The above program is linear in auditing policy. Therefore, there exists $\tilde{\rho}$ such that equation (8) evaluated at $\tilde{\rho}$ is a necessary and sufficient condition for the optimality of $\tau^*(l)$, on $l > d^*$. The multiplier $\tilde{\rho}$ is uniquely defined which implies that $\tilde{\rho} = \rho$ and that $\tau^*(l) = 1$ if and only if equation (8) is verified.

Using

$$\mathscr{R}(D; l) \equiv \int_0^D y h(y; l) dy + D(1 - H(D; l)),$$

the LHS of inequality (8) is given by:

$$Z(l) \equiv \int_{D_1^*(l)}^{D^*} \{\Psi^* - \Psi(y; l)\} h(y; l) dy.$$

This equals 0 at $D_1^*(l) = D^*$. What we need to prove is that it is increasing on $D_1^*(l) < D^*$. The derivative with respect to l is:

$$\begin{aligned} Z'(l) &= \int_{D_1^*(l)}^{D^*} \{\Psi^* h_l(y; l) + H_l(y; l)\} dy \\ &= \Psi^* \{H_l(D^*; l) - H_l(D_1^*(l); l)\} + \int_{D_1^*(l)}^{D^*} H_l(y; l) dy. \end{aligned}$$

Using $\Psi_y = -1 - \frac{h_y}{h} \Psi$ and integrating by part we obtain:

$$Z'(l) = \{\Psi^* - \Psi(D^*; l)\} H_l(D^*; l) + \int_{D_1^*(l)}^{D^*} \Psi(y; l) \left\{ h_l(y; l) - H_l(y; l) \frac{h_y(y; l)}{h(y; l)} \right\} dy.$$

The first term is positive when $D_1^*(l) < D^*$ since $\Psi_y < 0$ and $H_l > 0$. Moreover,

$$\frac{1}{h} \left\{ h_l - H_l \frac{h_y}{h} \right\} = \frac{\partial}{\partial y} \left(\frac{H_l}{h} \right) = -A_{ly} \geq 0.$$

Consequently, $Z'(l) > 0$ which shows that the optimal contract implies auditing above some level l^*, with $D_1^*(l^*) < D^*$. The value Ψ^* is then given by condition (7).

We have then characterized the optimal contract under the technology that assumes observability of (l, x) under bankruptcy. If we come back to the technology considered in our model, the contract cannot yield better surplus for the firm than the one described above. On the other hand, the contract just described does not depend upon the observability assumption, since it basically is a contingent debt contract. Moreover, its nature obviously make truthful revelation of accident damages optimal for the firm, even when l is not separately observed under bankruptcy. The contract above is therefore also the optimal contract under the weaker technology. □

B A non-expected utility model for insurance demand

In what follows, we will use D as a shortcut for $D(\Phi, b)$, keeping in mind that it is a function of Φ and b. Using (5), $V(\Phi, b)$ can be written as:

$$\begin{aligned} V(\Phi, b) &= (E_G - E_\Phi) - (\mathbf{E}_\Phi\left[R(D+l)\right] - E_\Phi) \\ &= E_G - E_F - I - b - c_0 \mathbf{E}_\Phi\left[G(D+l)\right]. \end{aligned}$$

$V(\Phi, b)$ defines Fréchet-differentiable, non-linear preferences on distributions of net losses. The induced local utility function $U_{\Phi b}(l)$ corresponds to the Fréchet derivative of V with respect to Φ.

Lemma 1. *The local utility function is given by:*

$$U_{\Phi b}(l) = K(\Phi, b)\left(R(D+l) - \frac{\mathbf{E}_\Phi[R'(D+l)]}{\mathbf{E}_\Phi[g(D+l)]} G(D+l)\right)$$

where $K(\Phi, b)$ is positive.

Proof. Consider a change $\Delta\Phi$ of Φ. Then, the corresponding change ΔD of the face value of the optimal debt contract can be computed to the first order:

$$\Delta D = \frac{\int_0^L \{R'(D+l) - c_0 g(D+l))\} \Delta\Phi(l) dl}{\mathbf{E}_\Phi\left[R'(D+l) - c_0 g(D+l)\right]}. \tag{9}$$

For the derivative of V, we obtain:

$$\begin{aligned} \Delta V &= -\mathbf{E}_\Phi\left[R'(D+l)\right] \Delta D + \int_0^L R'(D+l)\Delta\Phi(l) dl \\ &= \frac{\mathbf{E}_\Phi\left[c_0 g(D+l)\right]}{\mathbf{E}_\Phi\left[R'(D+l) - c_0 g(D+l)\right]} \\ &\quad \int_0^L \left\{ \frac{\mathbf{E}_\Phi\left[R'(D+l)\right]}{\mathbf{E}_\Phi\left[g(D+l)\right]} g(D+l) - R'(D+l)) \right\} \Delta\Phi(l) dl \end{aligned}$$

Integrating by part this formula provides the local utility function. $K(\Phi, b)$ is the term in factor of the integral, and it is positive as a consequence of the definition of $D(\Phi, b)$. □

Proof of Proposition 3

Proof. Consider a distribution Φ with at least two points in its support. Given that $R'(z) = 1 - G(z)$, $\psi(z) = \frac{R'(z)}{g(z)}$. It is then immediate that $U'_{\Phi b}(l) > 0$ if and only if:

$$\psi(D + l) > \frac{\mathbf{E}\left[\psi(D + l)g(D + l)\right]}{\mathbf{E}\left[g(D + l)\right]}.$$

Under the monotone likelihood ratio property, it follows that there exists a unique l_Φ such that $U_{\Phi b}$ is increasing below l_Φ and decreasing above. Moreover, when Φ admits a support with at least two points, it must necessarily be true that:

$$\inf_{l \in Supp\Phi} \psi(D + l) < \frac{\mathbf{E}\left[\psi(D + l)g(D + l)\right]}{\mathbf{E}\left[g(D + l)\right]} < \sup_{l \in Supp\Phi} \psi(D + l).$$

It follows that l_Φ must lie strictly between the lower bound and the upper bound of the support of Φ.

$U_{\Phi b}$ being strictly quasi-concave, there exists a mean preserving reduction in risk, around l_Φ, that is strictly preferred to Φ: take $\Phi + \lambda\Delta\Phi$, for λ small, the support of $\Delta\Phi$ is the same as Φ, and $\Delta\Phi(l) < 0$ if $l < l_\Phi$ and $\Delta\Phi(l) > 0$ if $l_\Phi < l$. Then $\int U_{\Phi b}(l)d\Delta\Phi(l) = -\int U'_{\Phi b}(l)\Delta\Phi(l)dl > 0$.

The set of distributions with support into $[0, L]$ and with a fixed mean μ is compact under the weak topology, while V is continuous on this set. The maximum of $V(\Phi, b)$ is thus be attained at some Φ^*. The above reasoning shows that Φ^* is the Dirac distribution concentrated at μ. In particular, for any non-Dirac distribution Φ, $V(\delta_{E_\Phi}, b) > V(\Phi, b)$.

Finally $U_{\Phi b}$ is concave for all Φ if and only if

$$\frac{\mathbf{E}_\Phi\left[R'(D + l)\right]}{\mathbf{E}_\Phi\left[g(D + l)\right]} g'(D + l) + g(D + l) > 0.$$

Hence the condition in the text. The result then follows from Machina (1982). □

Proof of Corollary 4

Denote by J the generic c.d.f. of a random variable $\widetilde{z}(l)$ such that $\widetilde{z}(l) = z(l)$ on $[0, L]\backslash W$. The set of such J is compact under the weak topology. Let J_W maximizes $V(J, b)$ on this set and $z_W(l)$ be the corresponding random variable. There exists l_W in the interior of the support of J_W, such that the local utility function is increasing below l_W and decreasing above. Suppose first that for all $l \in W$, $z_W(l) < l_W$. Then increasing slightly $z_W(l)$ would raise V, a contradiction. The same reasoning if $z_W(l) > l_W$ on W shows that $\inf_W\{z_W(l)\} \leq l_W \leq \sup_W\{z_W(l)\}$. If $z_W(l)$ is not constant on W, both inequalities are strict. Then increasing $z_W(l)$ for $l < l_W$ in W and reducing $z_W(l)$ for $l > l_W$, while still preserving the mean, induces a simple mean preserving reduction in risk around W and increases V. It follows that $z_W(l)$ must be constant on W. $\square$

C Proof of Proposition 4

We consider the following contract between the creditor and the firm: the creditor lends $I + \pi^*$ and audit occurs with probability 1; for a final cash flow y, the firm retains $\phi(y)$, where ϕ is concave and non-negative so as to satisfied limited liability.

Let $U(z)$ be the concave function defined by: $U(z) \equiv \mathbf{E}_G\left[\phi(x+z)\right]$. At the renegotiation stage, following the initial contract $\phi(.)$ and the insurance contract $r(.)$, the firm has an expected utility given by:

$$V = \int_0^L U(\pi^* - \pi - l + r(l))f(l)dl$$

where the premium is given by:

$$\pi = \mathbf{E}\left[r(l)\right] + c_1 \Pr\{r(l) > 0\}.$$

Since the creditor has the bargaining power, the firm cannot capture additional profit in the renegotiation stage and its insurance demand maximizes V. From Townsend (1979), the optimal insurance contract is a franchise contract with franchise $\bar{l}$ and deductible $\bar{d}$. We exhibit a financial contract such that i) the firm's demand for insurance is such that $\bar{l} = l^*$, $\bar{d} = d^*$, and ii) the insurance contract (l^*, d^*) leads to a value $V = V^*$ (the second-best expected profit).

Let us consider a contract of the form:

$$\phi(y) = \delta(a + by + \inf(y, M_0))$$

which leads to

$$U'(z) = \delta(b + G(M_o - z))$$

The firm's expected profit under a contract with franchise $\bar{l}$ and deductible $\bar{d}$ is:

$$\int_0^{\bar{l}} U(\pi^* - \pi - l)f(l)dl + \left(1 - F(\bar{l})\right) U\left(\pi^* - \pi - \bar{d}\right)$$

where the insurance premium is determined by:

$$\pi = \left(c_1 - \bar{d}\right)\left(1 - F(\bar{l})\right) + \int_{\bar{l}}^L lf(l)dl$$

Given the concavity assumption on $\phi(.)$, the contract (π^*, l^*, d^*) is optimal if and only if:

$$\begin{aligned}\int_0^{l^*} U'(-l)f(l)dl - U'(-d^*)F(l^*) &= 0\\ (l^* + c_1 - d^*)U'(-d^*) &= U(-d^*) - U(-l^*)\end{aligned}$$

which reduces to:

$$\int_0^{l^*} G(M_o + l)f(l)dl - G(M_o + d^*)F(l^*) = 0 \quad (10)$$

$$(l^* + c_1 - d^*)(b + G(M_o + d^*)) - \int_{d^*}^{l^*} G(M_o + l)dl = 0 \quad (11)$$

We first prove that (10) admits a solution.

Lemma 2. *There exists* $M_0,\ D^* < M_0 < \bar{x} - d^*$, *solution of (10).*

Proof. For $M_0 = \bar{x} - d^*$, the LHS of (10) is negative. We show that for $M_0 = D^*$ the LHS is positive. The result then follows by continuity.

From the FOC of Theorem 1 and using $\psi(x) = \frac{1-G(x)}{g(x)}$, we have:

$$\begin{aligned}B &\equiv \int_0^{l^*} G(D^* + l)f(l)dl - G(D^* + d^*)F(l^*)\\ &= -\psi(D^* + d^*)\left\{\int_0^{l^*} g(D^* + l)f(l)dl - g(D^* + d^*)F(l^*)\right\}\\ &= -\psi(D^* + d^*)\left\{\int_0^{l^*} \left\{\int_{d^*}^{l} g'(D^* + s)ds\right\} f(l)dl\right\}\\ &= -\psi(D^* + d^*)\left\{\int_{d^*}^{l^*} g'(D^* + l)(F(l^*) - F(l))dl - \int_0^{d^*} g'(D^* + l)F(l)dl\right\}\end{aligned}$$

Using the monotone likelihood ratio property, one can obtain the following inequality:

$$\begin{aligned}B &\geq -\psi(D^* + d^*)\frac{g'(D^* + d^*)}{g(D^* + d^*)}\left\{F(l^*)\int_{d^*}^{l^*} g(D^* + l)dl - \int_0^{l^*} g(D^* + l)F(l)dl\right\}\\ &\geq -\psi(D^* + d^*)\frac{g'(D^* + d^*)}{g(D^* + d^*)}B\end{aligned}$$

Since $\psi(x)$ is deceasing under the monotone likelihood ratio property,

$$-\psi(D^* + d^*)\frac{g'(D^* + d^*)}{g(D^* + d^*)} = \psi'(D^* + d^*) + 1 < 1$$

This implies that $B > 0$. $\square$

We can now conclude the proof of the proposition. Choose first M_0 and b_0 solutions of (10) and (11). Choose then a_0 such that the limited liability constraint is satisfied. Finally choose δ_0 such that

$$\delta_0 \left\{ a_0 + b_0 \mathbf{E}\left[x - l + r^*(l) \right] + \mathbf{E}\left[\inf\left\{ x - l + r^*(l), M_0 \right\} \right] \right\} = V^*$$

Then the initial contract is renegotiated to a debt contract with face value D^* once the firm has acquired the insurance contract (π^*, l^*, d^*). □

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