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Maximum likelihood estimation of deposit insurance value with interest rate risk

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Abstract

This paper develops a maximum likelihood estimation method for the deposit insurance pricing model of Duan, Moreau and Sealey (DMS) [J. Banking Financ. 19 (1995) 1091.]. A sample of 10 US banks is used to illustrate the estimation method. Our results are then compared to those obtained with the modified Ronn–Verma method used in DMS. Our findings reveal that the maximum likelihood method yields estimates for the deposit insurance value much larger than the ones based on the modified Ronn–Verma method. We conduct a Monte Carlo study to ascertain the performance of the maximum likelihood estimation method. The simulation results are clearly in favor of our proposed method.

JEL classification: C22
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1. Introduction

Option valuation models have been among the most important innovations in finance. As Black and Scholes (1973) pointed out, it is possible to view most

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assets as options. Merton (1977) showed that deposit insurance, a contract insuring the deposits of a deposit-taking institution, can be modeled as a put option on its assets. The value of the deposit insurance can then be calculated using the Black and Scholes (1973) option pricing model with specific modifications arising from the deposit insurance contract. Subsequent to Merton (1977), a large literature on deposit insurance has emerged, in part due to the US savings and loan debacle in the 1980s and early 1990s; for example, Merton (1978), McCulloch (1985), Ronn and Verma (1986), Kane (1987), Pennacchi (1987a,b), Duan and Yu (1994a,b), Duan et al. (1995), Nagarajan and Sealey (1995) and Schreiber (1997), among many others.

This paper is primarily concerned with the methodological aspect of the theoretical deposit insurance pricing model developed by Duan et al. (DMS) (1995), which extends Merton's (1977) model to incorporate stochastic interest rates. DMS's generalization is useful because their model explicitly accounts for the term structure effect. It thus allows for an assessment of the impacts of interest rate risk on the bank's assets, equity and deposit insurance. The current empirical methodology for implementing their model is not adequate, however. A brief background on empirical implementations of the Merton (1977) model helps the understanding of our contention.

In the literature, the empirical implementation of the Merton (1977) model mostly relies on an estimation method put forward by Ronn and Verma (1986). In essence, the Ronn–Verma (1986) method relies on two equations: one relating the equity value to the bank's asset value, and the other relating the equity volatility to the asset volatility. The equity value can be directly observed, but the equity volatility must be estimated. The sample standard deviation of equity returns is thus used as the equity volatility. The two-equation system is then used to solve for two unknown variables: the asset value and volatility. However, as argued in Duan (1994), the theoretical premise of Merton's (1977) deposit insurance pricing model implies stochastic equity volatilities. The Ronn–Verma (1986) estimation method, by assuming a constant equity volatility, is thus incompatible with the theoretical model. It thus yields inconsistent estimates and produces unreliable inference for the bank's asset volatility parameter and deposit insurance value.

In the empirical literature on deposit insurance, it has been rare to see studies that fully utilizes the distributional assumption embedded in the specification of the theoretical model. Yet, as noted in Lo (1986) and Duan (1994), it is possible to explicitly derive in many cases the limiting distribution for the price estimator of a derivative security by directly using the embedded distributional assumption that is an integral part of the model specification. In Lo (1986), the aim was to derive the maximum likelihood estimator for the derivative security price when the underlying asset price can be observed. In Duan (1994), on the other hand, the maximum likelihood estimation method was proposed to deal with the estimation problem where the instrument underlying the derivative security cannot be directly observed. As an application, Duan (1994) showed how the proposed method can be applicable to the Merton (1977) deposit insurance pricing model. Duan and Yu (1994b) used the same estimation method to study the deposit insurance value for a group of Taiwanese banks. They found significant and practical differences in the deposit insurance values obtained using the Ronn–Verma (1986) and maximum likelihood methods.

In the empirical part of DMS (1995), the Ronn–Verma (1986) method was generalized so that it can be applicable to their deposit insurance pricing model under stochastic interest rates. DMS (1995) then used the modified estimation method to obtain empirical estimates for a large sample of US banks, and to evaluate the interest rate risk exposure of both deposit taking institutions and the deposit insuring agent. Not surprisingly, the same criticism on the estimation method in principle applies to DMS's (1995) modified Ronn–Verma procedure. It is also conceivable that the empirical inconsistency in the case of the DMS (1995) model may be more severe because of the greater complexity of the underlying stochastic system induced by stochastic interest rates.

The objective of this paper is to develop an appropriate estimation method for the DMS (1995) deposit insurance model. We take the transformed-data perspective put forward in Duan (1994) to develop a two-step maximum likelihood estimation procedure for the DMS (1995) model. The maximum likelihood estimates are then compared to those obtained by employing the modified Ronn–Verma method. Although the maximum likelihood estimation method is theoretically superior due to its many desirable asymptotic properties, its actual performance in terms of the DMS (1995) deposit insurance pricing model can only be gauged with a Monte Carlo study. We thus also conduct a Monte Carlo simulation study to evaluate the quality of our proposed estimation method.

The remainder of this paper is organized as follows. Section 2 briefly presents the DMS (1995) model. Section 3 derives the likelihood function of this model and introduces our proposed estimation procedure. Section 4 describes the data set and the assumptions used in the empirical study, and presents the empirical findings. The Monte Carlo simulation study is presented in Section 5. Section 6 is used to examine the sensitivity of the empirical estimates to changes in the empirical assumptions. Section 7 contains the concluding remarks. Technical details are provided in Appendices A and B.

2. The DMS (1995) model—a review

As in Vasicek (1977), the DMS (1995) model assumes that the instantaneous interest rate is governed by the following mean–reverting stochastic process

$$\mathrm{d}r_t = q(m - r_t)\mathrm{d}t + \upsilon \mathrm{d}Z_{r_t}, \tag{1}$$

where r_t is the instantaneous risk-free rate of interest at time t; m is the long-run mean of the interest rate; υ is the volatility of the interest rate; q is a positive constant measuring the magnitude of the mean-reverting force; and Z_{r_t} is a Weiner process. Using the above process as the basis and with the assumption of a constant risk premium λ, Vasicek (1977) showed that the price of a default-free zero-coupon bond with \$1 face value and maturity of $T-t$ periods equals

$$P(r_t,t,T) = A(t,T)\mathrm{e}^{-B(t,T)r_t}, \tag{2}$$

where

$$A(t,T) = \mathrm{e}^{\gamma[B(t,T)-(T-t)]-\frac{\upsilon 2B2(t,T)}{4q}},$$

$$B(t,T) = \frac{1}{q}\left[1-\mathrm{e}^{-q(T-t)}\right],$$

$$\gamma = m + \frac{\upsilon\lambda}{q} - \frac{\upsilon^2}{2q^2}.$$

The DMS (1995) deposit insurance modeling setup follows that of Merton (1977). At time $t = 0$, the bank acquires an asset portfolio, V, and finances its assets with insured interest-bearing deposits with face value of F and maturing at T. The bank's asset value is assumed to follow a log-normal process given by

$$\frac{\mathrm{d}V_t}{V_t} = \mu\mathrm{d}t + \sigma_V\mathrm{d}Z_{V_t}, \tag{3}$$

where V_t is the value of bank assets at time t; μ is the instantaneous expected return on bank assets; σ_V is the total volatility of the bank's asset return; and Z_{V_t} is a Weiner process. The processes Z_{V_t} and Z_{r_t} are correlated with a correlation coefficient of η.

Let $X = Fe^{R(t,T)T}$ denote the equity holders' terminal obligation to depositors where R(t,T) is the time t yield to maturity of a default free bond with maturity T. Given the previous assumptions about the stochastic process for the instantaneous interest rate and the bank asset value, DMS (1995) showed that the market value of deposit insurance per dollar of insured deposits at time t is given by

$$I_t(V_t,r_t) = XP(r_t,t,T)\left[1-N(h_t-\delta_t)\right] - V_t\left[1-N(h_t)\right], \tag{4}$$

where

$$h_t = \frac{1}{\delta_t}\ln\left[\frac{V_t}{P(r_t,t,T)X}\right] + \frac{\delta_t}{2},$$

$$\delta_t^2 = \left(\phi_V^2\upsilon^2+\psi^2\right)(T-t) + 2\phi_V\upsilon^2\left\{\frac{T}{q}+\frac{1}{q^2}\left[\mathrm{e}^{-qT}-1\right]\right\}$$
$$+\upsilon^2\left\{\frac{T}{q^2}+\frac{2}{q^3}\left[\mathrm{e}^{-qT}-1\right]+\left[\frac{1-\mathrm{e}^{-2qT}}{2q^3}\right]\right\},$$

$$\phi_V = \frac{\sigma_V\eta}{\upsilon},$$

$$\psi = \sigma_V\left(1-\eta^2\right)^{\frac{1}{2}},$$

and $N(\cdot)$ denotes the standard normal cumulative distribution function. The parameter ϕ_V is interpreted in DMS (1995) as the instantaneous interest rate elasticity of the bank's assets because $\phi_V = ((\mathrm{Cov}(dV_t/V_t, dr_t))/(\mathrm{Var}(dr_t)))$, or the regression coefficient of the percentage change in the asset value on the change in the instantaneous interest rate. The parameter ψ is interpreted as the credit risk because it is the variability of the component of the asset return that is orthogonal to the interest rate risk.

Expressions for the bank's equity value, interest rate elasticity and standard deviation were derived in DMS (1995). Specifically, the bank's equity value at time t is

$$S_t = V_t N(h_t) - XP(r_t,t,T)N(h_t - \delta_t). \tag{5}$$

The bank's interest rate elasticity at time t is

$$\phi_{S_t} = \Omega_t[\phi_V + B(t,T)] - B(t,T), \tag{6}$$

where

$$\Omega_t = N(h_t)\frac{V_t}{S_t}.$$

Finally, the standard deviation of the bank's equity return at time t is given by

$$\sigma_{S_t} = \sqrt{\phi_{S_t}^2 \upsilon^2 + \Omega_t^2 \psi^2}. \tag{7}$$

Similar to the Merton (1977) model, difficulties arise in implementing the DMS (1995) model. The parameter values of the system must be estimated. Without direct observations of the instantaneous interest rate, r_t, and the bank's asset value, V_t, parameter estimates are hard to obtain. Even if we are able to obtain parameter estimates, the lack of values for the bank's assets and instantaneous interest rate can still prevent us from applying the model. To overcome these difficulties, DMS (1995) modified the estimation method developed by Ronn and Verma (1986) for the Merton (1977) model. Instead of using two equations, the modified Ronn–Verma method employs three equations given by Eqs. (5), (6) and (7). The additional equation is Eq. (6), which arises from considering stochastic interest rates (please refer to Appendix A for details).

Unfortunately, the Ronn–Verma (1986) method and its modified version in DMS (1995) are not developed from a consistent statistical framework. Their methods mix up the random variable(s) with the model parameter(s); for example, the equity volatility is, according to the pricing model, a random variable even though the asset volatility is a constant. The sample variance of equity returns thus cannot be a consistent estimator of the equity volatility. Such a criticism was first raised in Duan (1994), in which an alternative approach based on a transformed-data framework was proposed.

3. Likelihood function and estimation procedure

3.1. Likelihood function

In this section, we follow the transformed-data approach of Duan (1994) to develop an estimation procedure for the DMS (1995) model. In order to obtain the likelihood function for the DMS (1995) model, we need to relate the unobserved random variables to the observed random variables. If the relationship is a one-to-one differentiable transformation, the density function based on the observed variables is simply the density of the unobserved variables multiplied by the determinant of the inverse of the Jacobian corresponding to the transformation.[1] In the DMS (1995) model, the two unobserved random variables are the instantaneous interest rate, r_t and the bank's asset value, V_t. The first function defining the functional transformation is the Vasicek (1977) bond pricing function in Eq. (2), which relates r_t to some observed bond price. The second function is the equity valuation equation in Eq. (5) which links the unobserved bank asset value to the observed equity value.

Denote by $\mathbf{w}_t$ the 2×1 vector of unobserved variables at time t; that is $\mathbf{w}_t = [r_t, \ln V_t]'$. Define $\mathbf{y}_t$ as a 2×1 vector of observed variables at time t; that is $\mathbf{y}_t = [P(r_t,t,T), S_t]'$. Let $\boldsymbol{\theta}$ denote the vector containing the parameters associated with the stochastic processes postulated for the two unobserved variables; that is, $\boldsymbol{\theta} = [q, m, \upsilon, \lambda, \mu, \sigma_V, \eta]'$. The mapping between the observed and unobserved variables is written as

$$\mathbf{Y} = \mathbf{M}(\mathbf{W};\boldsymbol{\theta}),$$

where

$$\mathbf{Y} = [\mathbf{y}'_1, \ldots, \mathbf{y}'_n]',$$

$$\mathbf{W} = [\mathbf{w}'_1, \ldots, \mathbf{w}'_n]'.$$

Let $\mathbf{DY} = \frac{\partial \mathbf{Y}}{\partial \mathbf{W}}\big|_{\mathbf{W}=\mathbf{M}^{-1}(\mathbf{Y};\boldsymbol{\theta})}$. Given the above mapping, the log-likelihood function for $\mathbf{Y}$ can be written as (see Theorem 2.1 of Duan, 1994)

$$L(\boldsymbol{\theta};\mathbf{Y}) = L_w(\boldsymbol{\theta};\mathbf{M}^{-1}(\mathbf{Y};\boldsymbol{\theta})) + \ln(|\det\{\mathbf{DY}^{-1}\}|),$$

where $L_w(\boldsymbol{\theta};\mathbf{M}^{-1}(\mathbf{Y};\boldsymbol{\theta}))$ and $\ln(|\det\{\mathbf{DY}^{-1}\}|)$ denote, respectively, the log-likelihood function of $\mathbf{W}$ and the logarithm of the determinant of the Jacobian. In

[1] The Jacobian is a matrix of partial derivatives computed from the transformation relating the unobserved variables to the observed.

Appendix B, a simple analytical expression for the logarithm of the determinant of the Jacobian is derived

$$\ln\left(|\det\{\mathbf{DY}^{-1}\}|\right) = -\sum_{t=1}^{n} \ln\left(P(r_t,t,T)B(t,T)V_t N(h_t)\right).$$

An expression for $L_w(\boldsymbol{\theta};\mathbf{M}^{-1}(\mathbf{Y};\boldsymbol{\theta}))$ can be derived by noticing that the one-period transition density for both $\ln(V_t)$ and r_t are normally distributed. The density function for $\mathbf{W}$ is thus a multivariate normal distribution. More precisely, we deal with the normal transition densities of $\ln(V_t/V_{t-1})$ and r_t, and their relevant moments are

$$E_{t-1}\left[\ln\left(\frac{V_t}{V_{t-1}}\right)\right] = \mu s - \frac{1}{2}\sigma_V^2 s, \quad \mathrm{Var}_{t-1}\left[\ln\left(\frac{V_t}{V_{t-1}}\right)\right] = \sigma_V^2 s, \tag{8}$$

$$E_{t-1}(r_t) = m + (r_{t-1} - m)\mathrm{e}^{-qs}, \quad \mathrm{Var}_{t-1}(r_t) = \frac{\upsilon^2}{2q}(1-\mathrm{e}^{-2qs}), \tag{9}$$

and

$$\mathrm{Cov}_{t-1}\left[r_t,\ln\left(\frac{V_t}{V_{t-1}}\right)\right] = \phi\sigma_V\eta\sqrt{s}, \tag{10}$$

where s is the length, in units of time, of the discrete interval between two adjacent observations, η is the correlation between the Z_{r_t} and Z_{V_t}, and $\phi = \sqrt{\frac{\upsilon^2}{2q}(1-\mathrm{e}^{-2qs})}$.

Define $\mathbf{u}_t = \left[r_t,\ln\left(\frac{V_t}{V_{t-1}}\right)\right]'$ and $\hat{\mathbf{u}}_t(\boldsymbol{\theta}) = \left[\hat{r}_t(\boldsymbol{\theta}),\ln\left(\frac{\hat{V}_t(\theta)}{\hat{V}_{t-1}(\theta)}\right)\right]'$ where the elements of $\hat{\mathbf{u}}_t(\boldsymbol{\theta})$ are computed using the inverse transformations of Eqs. (2) and (5) evaluated at the parameter value $\boldsymbol{\theta}$. The logarithm of the full-information likelihood function for the DMS (1995) model can thus be written as

$$\begin{aligned} &L\left(\boldsymbol{\theta};P(\cdot),S_t,t=1,\ldots,n\right) \\ &\quad = -\frac{n-1}{2}\ln\|\boldsymbol{\Sigma}\| - \frac{1}{2}\sum_{t=2}^{n}\left[\hat{\mathbf{u}}_t(\boldsymbol{\theta}) - E_{t-1}(\mathbf{u}_t)\right]\boldsymbol{\Sigma}^{-1}\left[\hat{\mathbf{u}}_t(\boldsymbol{\theta}) - E_{t-1}(\mathbf{u}_t)\right]' \\ &\qquad -\sum_{t=2}^{n}\ln\left[P(r_t,t,T)B(t,T)V_t N(h_t)\right], \end{aligned} \tag{11}$$

where $E_{t-1}(\mathbf{u}_t)$ and $\boldsymbol{\Sigma}$, the covariance matrix of $\mathbf{u}_t$, are formed using the terms in Eqs. (8), (9) and (10). Note that we have dropped the likelihood associated with the first data point. The first data point for the equity value is only used to obtain the implied asset value, which is in turn used to define the transition density. Its omission is a necessity because there does not exist a stationary density for V_0. This log-likelihood function can be used to obtain the maximum likelihood parameter estimates. Let $\hat{\boldsymbol{\theta}}_n$ denote the maximum likelihood parameter estimator for $\boldsymbol{\theta}$ based on the sample size n. We invoke the standard asymptotic theory to come up with the following asymptotic distribution

$$\sqrt{n}\left(\hat{\boldsymbol{\theta}}_n - \boldsymbol{\theta}_0\right) \sim N(0,\mathbf{F}^{-1}), \tag{12}$$

where $\mathbf{F}$ is the asymptotic information matrix whose sample estimate is

$$\hat{\mathbf{F}}_n = -\frac{1}{n}\frac{\partial^2 L(\cdot)}{\partial\boldsymbol{\theta}_i\partial\boldsymbol{\theta}_j}|_{\theta=\hat{\theta}_n}, \tag{13}$$

and $\boldsymbol{\theta}_0$ stands for the true parameter vector. We will later verify by a Monte Carlo simulation analysis that the above distribution is indeed a suitable one for the maximum likelihood estimator.

Using the maximum likelihood estimate, it is also possible to calculate the estimates for the bank's asset value, $\hat{V}_t(\hat{\theta}_n)$, the deposit insurance premium, $\hat{I}_t(\hat{\theta}_n)$, the interest rate elasticities, $\hat{\phi}_{V_t}(\hat{\theta}_n)$, and the bank's credit risk, $\hat{\psi}(\hat{\theta}_n)$, for every time point. Because these quantities result from some differentiable transformations of the parameter estimate $\hat{\theta}_n$, their asymptotic standard errors can be computed using the following standard asymptotic result that will also be used as an approximation

$$\sqrt{n}\left(f\left(\hat{\boldsymbol{\theta}}_n\right) - f(\boldsymbol{\theta}_0)\right) \sim N\left\{0,\left(\frac{\partial f'}{\partial\boldsymbol{\theta}}\mathbf{F}^{-1}\frac{\partial f}{\partial\boldsymbol{\theta}}\right)|_{\theta=\hat{\theta}_n}\right\},$$

where $f(\hat{\theta}_n)$ is any one of the above four differentiable transformations.

Although directly optimizing the log-likelihood function in Eq. (11) looks like a sensible way of approaching the estimation problem, it is actually not an ideal approach in practice. First, the likelihood function is defined for the data set comprising one specific bank's equity value series and the common bond price series. If there are more than one bank, we will obtain more than one set of parameter values governing the common interest rate dynamic. One can, of course, think of expanding the log-likelihood function to include all banks in the sample to conduct a joint estimation. It is not practical, however, when there are many banks in the sample. The second problem is the difference in the time horizons for bond pricing and equity valuation. The Vasicek (1977) bond pricing model reflects the mean reversion in interest rates. The tendency for interest rates to return to their average is meant to be a long-run phenomenon. It is therefore reasonable to expect that the mean-reversion parameter can only be pinned down using a relatively long interest rate data series. The equity valuation model, on the other hand, depends on the bank's asset volatility parameter. Since the variation of the asset value under the diffusion specification is large, the volatility parameter can usually be estimated with precision using a relatively short time series. To allow for, say, year-to-year changes in the asset volatility, the use of an equity value time series shorter than the interest rate series may be desirable.

We thus devise a two-step estimation procedure. The first step estimates, through maximizing the likelihood function, the Vasicek (1977) bond pricing model parameters using interest rate data only. The likelihood function for this model is already given in Duan (1994), which is a special case of the likelihood function in Eq. (11). The second step estimates the asset value parameters with the likelihood function in Eq. (11) while fixing the interest rate parameters at the values obtained from the first step. This two-step procedure ensures the interest rate parameter estimates are the same for all banks in the sample. Moreover, it allows us to use a longer time series of interest rates to pin down the mean-reversion parameter for the interest rate dynamic. Since the parameters governing the asset value dynamic do not enter the bond pricing model, this two-step procedure continues to yield consistent parameter estimates. However, the estimation of the standard errors of the asset value parameters will be affected. Using the bond pricing parameter estimates as if they were the true parameter values does not account for the additional sampling errors brought about by the their estimation errors in the first step. To account for these sampling errors, the standard errors of the asset value parameters will be taken from the full Fisher information matrix in equation (13), where $\hat{\theta}_n$ is set to the parameter values obtained in the first and second step of the two-step estimation procedure. The statistical properties of our two-step procedure will be studied later using a Monte Carlo analysis.

3.2. Data set and simplifying assumption

Our empirical analysis uses a subset of banks in the DMS (1995) data set during the period from 1981 to 1989. Specifically, we take 10 large US commercial banks to perform the analysis.[2] The original data set used in DMS (1995) consists of 72 banks. However, since our purpose is to illustrate the use of our estimation technique and to examine its implications, a subset consisting of 10 banks suffices. The balance sheet and stock return data are the same as in DMS (1995), which were taken from the Quarterly Bank Compustat tapes and the CRSP Daily Return file. Our interest rate data differ from those in DMS (1995) because our maximum likelihood analysis, different from the heuristic approach of DMS (1995), needs a longer data series in order to pin down the interest rate parameters. We collect from the Wall Street Journal the daily prices of the Treasury bill with a

[2] These 10 banks are Bank New York, Bank of America, Bankers Trust New York, Chase Manhattan, Chemical, Citicorp, First Interstate, JP Morgan, NCNB, Security Pacific.

maturity closest to three months. The price used is the bid-ask price average. To ensure that we have 10 years worth of daily bond price data for the first year in our sample, the interest rate series starts in the beginning of 1972 and runs all the way to the end of 1989. The series of the total equity value is constructed as follows. For each quarter, the first observation is the equity value reported in the quarterly balance sheet The subsequent observations are obtained using the stock returns from the CRSP Daily Return file. The debt series are the level of domestic deposits plus international deposits, total borrowing and preferred shares from the Compustat data file. Since the balance sheet figures are available quarterly, the total debt series can only be updated quarterly.

For the deposit insurance model to be empirically tractable, some simplifying assumptions are needed. Our assumptions are identical to those made in DMS (1995). First, it is assumed that all debts have the same effective maturity, which is the next annual bank auditing time. In other words, T equals 1 year. As a logical consequence of this assumption, the bank's equity must also have a maturity equal to 1 year. As argued in Ronn and Verma (1986), although the two maturities are conceptually different, a rational investor should link the maturity of the call to the audit periodicity. At the time of audit, the bank is either closed and equity holders receive zero, or, if solvent, the equity value equals the difference between the asset value and the face value of debts. Therefore, the time of audit should be perceived by equity holders as the maturity date of their option. It should be noted, however, that bank deposits typically expire over a broad spectrum of maturities, this simplifying assumption which was first adopted in Merton (1977) may adversely affect the quality of the empirical estimates.

The second assumption concerns the evolution of the maturity of debt over a quarter. In Ronn and Verma (1986) and DMS (1995), the deposit insurance value is estimated on a quarterly basis, resetting the value of T to 1 year at the beginning of each quarter. This procedure thus implicitly assumes that, in purchasing deposit insurance, banks buy a put every quarter with a maturity of 1 year and that the debt is rolled over every quarter so that its maturity at the beginning of each quarter is 1 year. In order to be consistent with this implicit assumption, during the estimation, the maturity of the call option on the asset value will be set equal to 1 year at the beginning of each quarter, and will decrease gradually to approximately three quarters of a year at the end of each quarter. Later in this paper, we will check the sensitivity of the estimated deposit insurance value to a different maturity assumption.

Finally, it is assumed that the insuring agent enforces bank closures only after the bank has already attained a negative net worth, i.e., capital forbearance is extended to banks. This situation may arise for two reasons. First the insuring agent's closure rules depend on book value rather than market value. Second, because of bankruptcy costs related to bank closures, the insuring agent might be willing to allow a bank to continue operations in order to avoid or delay such costs. To account for this closure condition, the exercise price of the option in Eq.

(5) is adjusted by a factor of ρ where $0<\rho<1$. The expression for the equity value given by Eq. (5) is thus modified to

$$S_t = V_t N(h_t^*) - \rho X P(r_t,t,T) N(h_t^* - \delta_t), \tag{14}$$

where

$$h_t^* = \frac{1}{\delta_t}\ln\left[\frac{V_t}{\rho P(r_t,t,T)X}\right] + \frac{\delta_t}{2}.$$

As in Ronn and Verma (1986) and DMS (1995), a value of $\rho = 0.97$ will be used.[3] The sensitivity of the deposit insurance estimate to a change in ρ will also be analyzed later.

4. Empirical results

The two-step maximum likelihood estimation procedure is implemented as follows. In the first step, the Vasicek (1977) model is estimated with the daily interest rate series for the 10-year period preceding a particular year (inclusive), say, 1981. For the second step, the asset value parameters are estimated over a 1-year interval for, say, 1981. The same procedure is repeated for every year in our sample from 1981 to 1989.

Table 1 presents the estimation results for the Vasicek (1977) model. Since the first year used in our assessment of deposit insurance value is 1981, the interest rate data thus begin in 1972 to yield a 10-year sample. Similarly, the last interest rate data set used covers the 10-year interval from 1980 to 1989. In terms of the magnitude and statistical significance, the parameter estimates are similar to those for the Vasicek (1977) model reported in Duan (1994). It is worth noting that the estimates for both the interest rate volatility parameter and the risk premium parameter are fairly stable for different sample periods, but the estimates for the mean-reversion intensity parameter and the long-run average interest rate vary a great deal.

In the second step, maximization of the likelihood function in Eq. (11) is performed with respect to parameters: μ, σ_V and η, using the estimates for m, q, υ and λ obtained in the first step. Table 2 presents the results for one specific bank —Citicorp. Since the results are of the same nature for all the banks in our sample, we present the average results for the 10 banks in Table 3 instead of reporting the detailed results for individual banks. In these tables, equity values are year-end market values in million dollars. Debt values are year-end book values in million

[3] In keeping with the modified Ronn–Verma method used in DMS (1995), the parameter ρ is not treated as a free parameter in the likelihood function, although the maximum likelihood method can easily allow it to be a free parameter. If this parameter was included in the likelihood function, the modified Ronn–Verma method in DMS (1995) would need a fourth equation to pin down four unknowns.

Table 1
Estimation results for the Vasicek (1977) bond pricing model using daily data of the 3-month US Treasury yield

Parameters	Years								
	1972–1981	1973–1982	1974–1983	1975–1984	1976–1985	1977–1986	1978–1987	1979–1988	1980–1989
m	0.1442	0.0869	0.0834	0.0833	0.0958	0.0914	0.1055	0.1085	0.0644
	(0.1357)	(0.0551)	(0.0586)	(0.0681)	(0.0852)	(0.0778)	(3.7592)	(0.1072)	(0.0195)
q	0.1035	0.2179	0.2055	0.1735	0.1320	0.1471	0.0026	0.1090	0.5314
	(0.1656)	(0.1883)	(0.1851)	(0.1768)	(0.1792)	(0.1817)	(0.0000)	(0.2440)	(0.2607)
v	0.0360	0.0344	0.0338	0.0326	0.0318	0.0319	0.0307	0.0294	0.0319
	(0.0007)	(0.0008)	(0.0007)	(0.0007)	(0.0007)	(0.0007)	(0.0004)	(0.0010)	(0.0011)
λ	2.249	2.894	2.815	2.8962	2.4784	2.4853	2.2360	2.256	3.9170
	(0.5887)	(0.6163)	(0.6182)	(0.6180)	(0.6159)	(0.6183)	(0.5625)	(0.5228)	(0.5103)

Table 2
The maximum likelihood estimates and the modified Ronn-Verma estimates of the DMS (1995) deposit insurance pricing model for Citicorp

Year end	1981	1982	1983	1984	1985	1986	1987	1988	1989
Equity	3164	4135	4625	4893	6388	7294	5890	8256	9377
Debt	108,359	115,768	120,608	136,830	160,449	183,517	191,186	195,536	218,788
ψ_{mle}	0.290	0.188	0.090	0.131	0.058	0.079	0.069	0.078	0.069
	(0.024)	(0.014)	(0.005)	(0.012)	(0.002)	(0.004)	(0.003)	(0.005)	(0.004)
$\phi_{V\mathrm{mle}}$	−0.936	−0.576	−2.477	−0.616	−2.721	−3.349	−0.484	−2.952	1.110
	(0.994)	(1.065)	(0.838)	(1.919)	(0.520)	(0.706)	(0.246)	(0.627)	(0.345)
V_{mle}	82,991	102,760	115,807	128,576	156,939	175,524	187,132	190,116	216,381
	(2522)	(1552)	(2009)	(1474)	(1731)	(3858)	(372)	(2320)	(1217)
$\mathrm{IPP}_{\mathrm{mle}}$	2589	1408	672	867	487	726	396	580	397
	(236)	(138)	(181)	(114)	(122)	(227)	(23)	(131)	(65)
ψ_{mrv}	0.009	0.016	0.011	0.006	0.006	0.007	0.035	0.009	0.000
$\phi_{V\mathrm{mrv}}$	−0.968	−0.899	−1.047	−1.060	−1.161	−1.057	−0.878	−0.979	−0.024
V_{mrv}	108,190	116,347	121,542	137,552	161,942	185,256	190,267	197,909	221,606
$\mathrm{IPP}_{\mathrm{mrv}}$	93	74	66	67	60	49	180	33	17

Table 3
The 10-bank averages of the maximum likelihood estimates and those of the modified Ronn–Verma estimates for the DMS (1995) deposit insurance pricing model

Year end	1981	1982	1983	1984	1985	1986	1987	1988	1989
Equity	1508	1670	1955	2140	2930	3407	2592	3597	4700
Debt	49,335	52,640	53,879	58,328	64,431	71,131	73,384	77,793	86,095
ψ_{mle}	0.289	0.207	0.136	0.235	0.115	0.240	0.165	0.269	0.158
$\phi_{V\,\mathrm{mle}}$	−0.836	−0.535	−4.276	−2.159	−4.638	−12.371	−0.528	−17.337	2.329
V_{mle}	38,190	44,842	47,405	49,646	59,566	55,783	65,997	62,728	79,741
$\mathrm{IPP}_{\mathrm{mle}}$	2470	1678	1396	1854	1142	2407	1291	2863	1330
ψ_{mrv}	0.008	0.012	0.009	0.007	0.008	0.011	0.036	0.009	0.013
$\phi_{V\,\mathrm{mrv}}$	−0.956	−0.889	−0.879	−1.066	−1.098	−0.990	−0.882	−1.032	−0.335
V_{mrv}	49,328	52,687	54,195	58,681	65,393	72,363	73,331	79,044	88,212
$\mathrm{IPP}_{\mathrm{mrv}}$	78	82	49	60	41	39	157	35	12

dollars from balance sheets. ψ_{mle} and $\phi_{V\text{mle}}$ are the maximum likelihood estimates of the instantaneous credit risk and instantaneous interest rate elasticity of bank assets on an annualized basis.[4] V_{mle} and IPP_{mle} are the maximum likelihood estimates for the year-end asset value and the insurance premium per dollar of insured deposits (in basis points). The standard errors of estimates are reported in parentheses. These tables also report ψ_{mrv}, $\phi_{V\text{mrv}}$, V_{mrv} and IPP_{mrv}, which are the estimates obtained from the modified Ronn–Verma method in DMS (1995). For both the maximum likelihood and modified Ronn–Verma methods, the parameter controlling the slack before closure, ρ, is set equal to 0.97. Since the deposit insurance contract is not dividend protected, the estimates of IPP are calculated using Eq. (4) adjusted for dividends.

The maximum likelihood estimates of the instantaneous interest rate elasticity of bank assets, $\phi_{V\text{mle}}$, are of a similar magnitude when compared to those obtained from the modified Ronn–Verma method. In the case of Citicorp, the 95% confidence interval around the maximum likelihood estimate contains, in many cases, the modified Ronn–Verma estimate. Most parameter estimates are negative, which is consistent with a negative correlation typically expected between asset value and interest rate. However, the estimates are statistically insignificant for the first half of the decade.

The estimates for the credit risk parameter, ψ_{mle}, are much higher when compared to the modified Ronn–Verma estimates, which is true for Citicorp as well as for the average of 10 banks. In the case of Citicorp, all estimates are statistically significant from zero using the usual significance level. All of the modified Ronn–Verma estimates fall way outside of their respective 95% confidence intervals around the maximum likelihood estimates. It is interesting to observe that the difference between two estimation methods can be so drastic in terms of the credit risk parameter. The modified Ronn–Verma method essentially forces the stochastic variables such as the equity's interest rate elasticity and equity volatility to be constant. Its numerical effects are, however, different in relative magnitude for the asset's interest rate elasticity and for the asset's credit risk. Intuitively, one may view this as a result due to less variability in the equity's interest rate elasticity.[5] Consequently, erroneously treating it as a constant, as in the modified Ronn–Verma method, has a smaller numerical effect.

The maximum likelihood estimates for the year-end bank asset value are generally lower than those obtained using the modified Ronn–Verma method. Again, the statement is true for Citicorp and for the average of 10 banks. For

[4] Although maximization is performed with respect to μ, σ_V and η, the values reported in the tables are the estimates of ϕ_V and ψ. This simplifies our comparison with the modified Ronn–Verma methodology which directly obtains estimates for ϕ_V and ψ.

[5] In accordance with the formula in Eq. (6), the equity's interest rate elasticity is stochastic because Ω_t is stochastic. However, its multiplier which is the interest rate elasticity gap, $\phi_V + B(t,T)$, is usually small for banks. As a result, the equity's interest rate elasticity, although stochastic, is not expected to vary much.

Citicorp, the modified Ronn–Verma estimate of the bank asset value fall outside of the 95% confidence interval around the maximum likelihood estimate. In fact, the maximum likelihood estimates are, in most cases, below the book values of debts, effectively making equity an out-of-the-money call option.

The maximum likelihood estimates of IPP are much higher than those from the modified Ronn–Verma method for Citicorp and for the average of 10 banks. None of the modified Ronn–Verma insurance premium estimates falls within its corresponding 95% confidence interval around the maximum likelihood estimate. These deposit insurance premium estimates are considerably higher than the actual rate charged by the insuring agency over the sample period. The rate levied by the Federal Deposit Insurance Corporation over the sample period was around 12 basis points. This official rate was significantly lower than the market value computed according to the DMS (1995) model in all sample years in the case of Citicorp.

A higher insurance premium rate obtained by the maximum likelihood method is caused by two factors. First, comparing to the modified Ronn–Verma estimate, the maximum likelihood method yields a higher credit risk, which in turn increases the value of the deposit insurance (a put option). Second, unlike the modified Ronn–Verma method, the maximum likelihood estimates for the asset value are in all cases lower than the corresponding book values of debts. Since the debt value serves as the strike price for the put option, it results in a higher insurance premium rate.

The large difference between the maximum likelihood and modified Ronn–Verma estimates for the deposit insurance premium rate is consistent with the result reported in Duan and Yu (1994b), in which the maximum likelihood estimates for the Merton (1977) model were compared to the estimates obtained from the Ronn–Verma (1986) method. The maximum likelihood estimates of IPP reported in that study were found to be consistently larger than those obtained from the Ronn–Verma (1986) method.

5. A Monte Carlo study

In this section, we conduct a Monte Carlo analysis to assess the quality of the two-step estimation procedure. The data for the Monte Carlo study are simulated as follows.

(1) Let r_t^* and V_t^* denote the simulated values of the instantaneous interest rate and asset value at time t. Using the transition density functions corresponding to Eqs. (1) and (3). Specifically, r_t^* and V_t^* are simulated according to the dynamics

$$r_t^* = m + (r_{t-1}^* - m)\mathrm{e}^{-qs} + \phi\epsilon_{r_t},$$

$$V_t^* = V_{t-1}^*\exp\left(\mu s - 0.5\sigma_V^2 s + \sigma_V\sqrt{s}\,\epsilon_{V_t}\right),$$

where ϵ_{r_t} and ϵ_{V_t} are standard normal random variables with correlation η, $\phi = \sqrt{\frac{v^2}{2q}(1-e^{-2qs})}$, $r_0^* = m$ and $V_0^* = 100{,}000$. To be consistent with the use of daily data, we set $s = 1/252$. The annualized parameter values used in the equations above are: $m = 0.1$, $q = 0.2$, $v = 0.03$, $\lambda = 2.0$, $\mu = 0.05$, $\sigma_V = 0.05$ and $\eta = -0.5$.

(2) Use these simulated time series and Eqs. (2) and (5) to compute the corresponding series for the simulated bond and equity prices. The bond prices are computed using the maturities corresponding to the interest rate data set of the empirical section, which is roughly 3 months. As in the empirical analysis, the book value of debts is fixed over any quarter. The debt value is set to 90,000 for the first quarter. Each subsequent quarter the debt is increased by 2000. The maturity used to compute the equity value is set to 1 year at the beginning of a quarter, and then decreases gradually over the quarter.

Our Monte Carlo simulation results for the two-step maximum likelihood method are reported in Table 4. These results are obtained by repeating the simulation and estimation for 500 times. As in our empirical analysis, the experiment is performed using different sample sizes for interest rates and equity values. For the Vasicek (1977) model, 2520 data points are used, which corresponds to 10 years worth of daily data. The sample size for the bank's equity value is set to 252 data points, which corresponds to 1 year worth of daily data.

Rows one, two, three and four of Table 4 report the true parameter values, the medians, the means and the standard deviations of the parameter estimates, respectively, for ϕ_V, ψ, $(V_T - \hat{V}_T)$ and $(IPP_T - \widehat{IPP}_T)$. Note that V_T and IPP_T are, respectively, the simulated asset value and the computed deposit insurance value using the simulated asset value and the true parameter values. $\hat{V}_T$ and $\widehat{IPP}_T$ are the corresponding estimates using the maximum likelihood estimation method. Since both the simulated asset value and the computed insurance premium rate are different in each replication, it is more meaningful to report the differences. If the

Table 4
A Monte Carlo analysis of the two-step maximum likelihood estimation procedure for the DMS (1995) deposit insurance pricing model (500 repetitions)

	ϕ_V	ψ	$IPP_T - \widehat{IPP}_T$	$V_T - \hat{V}_T$
True value	0.8333	0.0433		
Median	−0.7312	0.0434	−0.0133	0.1306
Mean	−0.7317	0.0434	−0.5625	5.5129
Std. dev.	0.1314	0.0027	12.5169	122.6652
25% cov. rate	0.2880	0.2560	0.2400	0.2400
50% cov. rate	0.4920	0.4600	0.4760	0.4760
75% cov. rate	0.7340	0.7280	0.7700	0.7700
95% cov. rate	0.9140	0.9260	0.9260	0.9260

estimation method is good, these differences should be centered around zero. As the results indicate, the parameter estimate for ψ is indeed nicely centered at its true value. The parameter estimate for ϕ_V slightly underestimates (in magnitude) the true value of the parameter. Our method overestimates the deposit premium rate by approximately 0.56 basis points, but underestimates the asset value by approximately 0.01%. Note that the biases are statistically insignificant by the usual standard except for ϕ_V; for example, the difference in two asset values has a standard deviation of 122.6652, which can be translated into a standard deviation of $5.49(122.6652/\sqrt{500})$ for the mean value.

The remaining four rows of Table 4 report the probability that the true parameter lies in the α% confidence interval. This probability is referred to as the α% coverage rate. For example, to obtain a 95% coverage rate, $\text{Prob}\{|\theta_i - \hat{\theta}_{iT}| < 1.96 \times \text{s.e.}(\hat{\theta}_{iT})\}$ is computed, where θ_i denotes the ith parameter of the model and $\text{s.e.}(\hat{\theta}_{iT})$ represents the estimated standard error for the ith parameter estimator. The coverage rates also indicate that, for this sample size of 252, the normal distribution is a reasonable approximation to the sampling distribution of the estimator.

6. Sensitivity to changes in assumptions

Our estimates for the DMS (1995) model were obtained under some specific simplifying assumptions. In this section, we examine the sensitivity of the estimates to changes in some of these assumptions. Specifically, we study the assumption on the closure rule and maturity of the bank equity (viewed as a call option).

6.1. Closure rule

Although it is reasonable to assume that the insuring agent closes a bank only after the bank has attained a negative net worth, the percentage of the debt buffer is somewhat arbitrary. Ronn and Verma (1986) used $\rho = 0.97$ because it yielded an aggregate market value based deposit insurance premium equivalent to the actual amount charged by the FDIC. Clearly this rationale cannot apply to our setting, in which the estimated deposit insurance values are found to be much higher than those based on the modified Ronn–Verma method. It is intuitive to reason that a higher ρ causes both the equity and deposit insurance values to decrease because less slack is extended to the equity holder. This intuition is right, however, only when the bank asset value and other model parameters remain unchanged. The situation facing analysts when ρ is altered is very different, however. In reality, the observed equity value series stay fixed even if we change this assumption. Such a change actually forces the estimated bank asset value and the model parameters to adjust.

Table 5
Sensitivity of the maximum likelihood estimates for the DMS (1995) deposit insurance pricing model to changes in assumptions

Year end	1981	1982	1983	1984	1985	1986	1987	1988	1989
Equity	1508	1670	1955	2140	2930	3407	2592	3597	4700
Debt	49,335	52,640	53,879	58,328	64,431	71,131	73,384	77,793	86,095
Decreasing maturity of the call option (equity) $\rho = 1$									
ψ_{mle}	0.292	0.209	0.138	0.237	0.116	0.242	0.166	0.271	0.160
$\phi_{V\,\mathrm{mle}}$	−0.836	−0.529	−4.335	−2.139	−4.679	−12.479	−0.504	−17.489	2.377
V_{mle}	39,049	45,941	48,548	50,877	61,107	57,199	67,760	64,253	81,713
$\mathrm{IPP}_{\mathrm{mle}}$	2352	1534	1274	1731	1023	2304	1156	2769	1215
Constant maturity of the call option (equity) $\rho = 0.97$									
ψ_{mle}	0.157	0.148	0.103	0.176	0.148	0.201	0.140	0.150	0.108
$\phi_{V\,\mathrm{mle}}$	0.052	0.029	−5.099	−2.922	−8.166	−11.579	1.945	−11.567	−1.777
V_{mle}	45,568	49,701	49,189	52,986	55,162	53,143	66,890	66,954	82,410
$\mathrm{IPP}_{\mathrm{mle}}$	1225	1092	1448	1515	1991	3136	1215	2256	878

We set $\rho = 1$ to study the effect of increasing ρ. The assumption on maturity of equity (as a call option) continues to require a decreasing maturity within a quarter but subject to a quarterly reset to 1 year. The 10-bank averages are reported in the second panel of Table 5. Comparing these results with those in Table 3, the estimates for the model parameters do not change by much, but the estimated bank asset values become larger. A larger bank asset value in turn decreases the deposit insurance (put option) value because the book value of debts (strike price) minus the bank asset value decreases. Our results for IPP indeed support such a reasoning.

6.2. Maturity of the call option

The assumption examined here is concerned with the maturity of equity (a call option on the bank's assets). There is no a priori reason to believe that equity holders perceive a decreasing maturity over a quarter. This implicit assumption made in Ronn and Verma (1986) and DMS (1995) has more to do with the practical consideration concerning the frequency of the available balance sheet data rather than describing the actual behavior of equity holders.

We now consider an alternative assumption which postulates a constant maturity (1 year) over the whole sample. This assumption was adopted in Duan (1994) and Duan and Yu (1994b) where the maximum likelihood estimates for the Merton (1977) model were obtained. The third panel of Table 5 reports the 10-bank averages based on this alternative assumption. As the results indicate, the change in the maturity assumption produces changes in parameter values, asset values and insurance premium rates when they are compared to Table 3. The estimated deposit insurance premium rates in some years are lower than those under the assumption of decreasing maturity. However, their values are still very high compared to the actual rate charged by the FDIC or the estimates obtained using the modified Ronn–Verma method.

7. Conclusion

In this paper, a two-step estimation methodology for the DMS (1995) deposit insurance pricing model is developed. The estimation method relies on the full-information likelihood function constructed from the model, and thus enjoys all the benefits associated with maximum likelihood estimation. We carry out a Monte Carlo study to examine the performance of the proposed method for the typical sample size used in deposit insurance pricing. Our results suggest that the method, although relying on asymptotic inference, performs satisfactorily for our sample size.

Our empirical study of 10 US banks reveals that the earlier results obtained in DMS (1995) are questionable. Their empirical estimates were obtained in a way that is inconsistent with their theoretical model, and the magnitude of their deposit insurance value estimate is found to be much smaller than what their theoretical model really suggests. The deposit insurance value estimate is also shown to be sensitive to the assumptions made for the implementation of the theoretical model. This suggests that one must treat simplifying assumptions with care in future empirical studies of deposit insurance pricing.

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Appendix A. The modified Ronn–Verma estimation method

In DMS (1995), the Ronn–Verma (1986) method was modified to obtain the estimates for the asset value V, the interest rate elasticity of the asset ϕ_V, and the credit risk ψ. They rely on a three-equation system as opposed to the two-equation system of Ronn–Verma (1986). The additional equation is due to their considera-tion of interest rate elasticity. Specifically, the three-equation system is as follows

$$S_t = V_t N(h_t^*) - \rho X P(r_t,t,T) N(h_t^* - \delta_t),$$

$$\phi_{S_t} = \Omega_t[\phi_V + B(t,T)] - B(t,T),$$

$$\sigma_{S_t} = \sqrt{\phi_{S_t}^2 v^2 + \Omega_t^2 \psi^2},$$

where the values for S_t, ϕ_{S_t} and σ_{S_t} are obtained from the equity value and interest rate data. The standard deviation of the equity return for a given year, σ_{S_t}, is calculated as the sample standard deviation of all daily returns during the last quarter of the year. The year-end interest rate elasticity of equity, ϕ_{S_t}, is calculated from a linear regression of the daily equity return on the change in the daily 3-month Treasury bill rate over the last quarter of the year. For the interest rate parameters, we implement their method differently. DMS (1995) used ad hoc values for the interest rate parameters. Since we have the maximum likelihood parameter estimates readily available in this paper, we have used such estimates instead. Note that the additional parameter ρ is used to capture capital forbearance which is a result of the generalization introduced in Section 3.2.

Appendix B. The Jacobian term in the likelihood function

DY is a matrix of dimension $2n \times 2n$ where n is the number of data points[6]; that is,

$$\mathbf{DY} = \begin{bmatrix} \mathbf{dy}_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \mathbf{dy}_n \end{bmatrix},$$

and

$$\mathbf{dy}_t = \begin{bmatrix} \dfrac{\partial P(r_t t,T)}{\partial r_t} & \dfrac{\partial P(r_t,t,T)}{\partial \ln V_t} \\ \dfrac{\partial S_t}{\partial r_t} & \dfrac{\partial S_t}{\partial \ln V_t} \end{bmatrix}.$$

Since the mapping between the observed and unobserved variables does not involve lagged values, the matrix of partial derivatives is block diagonal with n blocks of dimension 2×2. The inverse of this matrix can be obtained by separately inverting the n matrices of dimension 2×2 found on the diagonal. The determinant of this inverse is simply the product of the determinants of these n matrices.

The individual elements in each of the n matrices are given by

$$\frac{\partial P(r_t,t,T)}{\partial r_t} = -P(r_t,t,T)\,B(T-t),$$

$$\frac{\partial P(r_t,t,T)}{\partial \ln V_t} = 0,$$

$$\frac{\partial S_t}{\partial r_t} = \left[V_t N(h_t) - S_t\right] B(T-t),$$

$$\frac{\partial S_t}{\partial \ln V_t} = V_t N(h_t).$$

[6] Note that one of the unobserved variables can be defined as the logarithmic asset value instead of the asset value itself. Its required derivatives, according to Duan (1994, Theorem 2.1), thus becomes $((\partial S_t)/(\partial \ln V_t))$. This approach is equivalent to the correction, which is described in Duan (2000), to the deposit insurance application originally stated in Duan (1994).

Using the above results and the well-known expression for the inverse of the 2×2 matrix, the determinant of the inverse of $\boldsymbol{dy}_t$ is found to be

$$\det\left\{\mathbf{dy}_t^{-1}\right\} = -\frac{1}{P(r_t,t,T)\,B(t,T)V_t N(h_t)}.$$

The Jacobian can thus be written as

$$\prod_{t=1}^{n}\left|\frac{1}{P(r_t,t,T)\,B(t,T)V_t N(h_t)}\right|.$$

Taking the logarithm of the above expression and noticing that $P(r_t,t,T)$,$B(t,T)$,V_t and $N(h_t)$ can only take positive values give rise to the logarithm of the Jacobian term as

$$\ln\left(\left|\det\left\{\mathbf{DY}^{-1}\right\}\right|\right) = -\sum_{t=1}^{n}\ln\left(P(r_t,t,T)\,B(t,T)V_t N(h_t)\right).$$

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