DIFFIDENCE THEOREM, STATE DEPENDENT PREFERENCES AND DARA.

by

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Abstract

Gollier and Kimball (1994,1996) have developed a standard technique based on the diffidence theorem. This theorem provides a very simple instrument to solve relatively sophisticated problems when preferences are state independent. The object of this article is to show that their theorem is also very useful to derive significant results with state dependent preferences. Using the reference set notion and an extension of the diffidence theorem, we establish formally necessary and sufficient conditions on the reference set in order to obtain prudence and decreasing absolute risk aversion.

Keywords: State Dependent Preferences, Reference Set, Diffidence Theorem, Prudence, Decreasing Absolute Risk Aversion (DARA).

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1 Introduction

The expected utility theory has been developed for state independent utility functions, and then extended to the state dependent world. Some authors have analyzed situations related to state dependent preferences. Among them, Dehez and Drèze(1982) and Shavell(1978) analyzed individual demand for insurance and safety; Drèze(1986,1987), Karni(1985) and Rustichini and Drèze (1994) studied the foundations of state dependent preferences, while Dionne(1982) considered moral hazard in insurance contracting (see also Dionne (1990)).

Edi Karni(1983,1985) has introduced the reference set notion in order to compare individuals' risk attitudes when preferences are state dependent. Karni also defined the risk premium function in that case and proposed a prerequisite for the definition of decreasing absolute risk aversion which is the autocomparability condition. This condition is claimed to be equivalent to the linearity of the reference set (See also Roëll (1984) for a discussion on this condition).

Gollier and Kimball (1994,1996) have developed a standard technique to analyse behavior under uncertainty, based on the diffidence theorem. This theorem provides a very simple instrument to solve relatively sophisticated problems. It doesn't only allow to systemize the way by which existing concepts of comparing risk attitudes are characterized, but also permits the definition of new concepts.

In this article, using the reference set notion and an extension of the diffidence theorem to state dependent preferences, we establish formally necessary and sufficient conditions on the reference set in order to obtain prudence and decreasing absolute risk aversion. We show that the linearity of the reference set is not necessary to obtain the results. Consequently, the prerequisite proposed by Karni is sufficient but not necessary for DARA.

2 The Reference Set Concept

Suppose that preferences can be represented by a von Neumann-Morgenstern utility function of final wealth $U(w)$ and that there exists two states of nature $i = s, t$. We denote by S the set of states of nature and by W the set of final wealths. We consider a risk averse individual with state dependent utility functions $U_i(w_i)$ that are strictly increasing and strictly concave $\forall i \in S$. We use p_s and $p_t = 1 - p_s$ for the probabilities of the risky situation, and w_s and w_t for the corresponding final wealths.

This means that the expected wealth is equal to

$$E(w_i) = p_s w_s + p_t w_t, \tag{1}$$

where E is the expectation operator. The expected utility is equal to

$$EU_i(w_i) = p_s U_s(w_s) + p_t U_t(w_t). \tag{2}$$

The reference set methodology supposes that every rational individual facing a choice problem under uncertainty will maximise his expected utility (2) by choosing optimal values for w_s and w_t subject to the expected wealth constraint (1). At the optimum, all the individual choices are in the reference set, defined by:

$$U_i^{'}(w_i^*) = \lambda \qquad \forall\, i \in S \tag{3}$$

where λ is the Lagrangian multiplier for constraint (1) and $U_i^{'}(w_i^*) > 0$ is the marginal utility in state i.
Notice that under risk aversion, $\forall\, i \in S$ (Karni, 1990):

$$U_i^{''}(w_i^*) < 0. \tag{4}$$

From (3), the reference set function is then given by:

$$w_t^* = (U_t^{'})^{-1}(U_s^{'}(w_s^*)) = f_t(w_s^*) \tag{5}$$

where s is taken here as the reference state of nature. Consequently, we can write

$$U_s^{'}(w_s^*) = U_t^{'}(f_t(w_s^*)).$$

From this reference set expression, we establish the following relations:

$$U_s^{''}(w_s^*) = U_t^{''}(w_t^*) f_t^{'}(w_s^*) \tag{6}$$

$$U_s^{'''}(w_s^*) = U_t^{'''}(w_t^*) f_t^{'2}(w_s^*) + U_t^{''}(w_t^*) f_t^{''}(w_s^*) \tag{7}$$

so that we can define from (6) and (7), and by using (3) and (4),

$$A_s(w_s^*) = A_t(w_t^*) f_t^{'}(w_s^*) \tag{8}$$

and

$$P_s(w_s^*) = P_t(w_t^*) f_t^{'}(w_s^*) - \frac{f_t^{''}(w_s^*)}{f_t^{'}(w_s^*)} \tag{9}$$

where $A_i(w_i^*)$ measures the absolute risk aversion in the i^{th} state of nature and $P_i(w_i^*)$ measures the prudence concept in the i^{th} state of nature, both in the reference set[1].

[1] See Kimball (1990) for the definition of prudence and Eeckhoudt & Kimball (1992) for an application to insurance.

3 Diffidence Theorem and State Dependent Preferences.

Developed for state independent preferences, the diffidence theorem can be extended to state dependent preferences by introducing the random variable $\tilde{x}_i$. Then, for a given value x_0, the theorem can be stated as having the following relationships[2]:

$$\forall (w_s^*, w_t^*), \tilde{x}_i : Eg_{1i}(w_i^*, \tilde{x}_i) \leq Eg_{1i}(w_i^*, x_0) \Rightarrow Eg_{2i}(w_i^*, \tilde{x}_i) \leq Eg_{2i}(w_i^*, x_0)$$

if the following necessary and sufficient condition is respected in each state of nature:

$$\forall w_i^*, x_i : g_{2i}(w_i^*, x_i) - g_{2i}(w_i^*, x_0) \leq \frac{\frac{\partial g_{2i}(w_i^*, x_0)}{\partial x_i}}{\frac{\partial g_{1i}(w_i^*, x_0)}{\partial x_i}} [g_{1i}(w_i^*, x_i) - g_{1i}(w_i^*, x_0)],$$

with the following necessary conditions[3]:

$$NC1 \qquad \forall w_i^* : \frac{\frac{\partial g_{2i}(w_i^*, x_0)}{\partial x_i}}{\frac{\partial g_{1i}(w_i^*, x_0)}{\partial x_i}} \geq 0$$

$$NC2 \qquad \forall w_i^* : \frac{\partial^2 g_{2i}(w_i^*, x_0)}{\partial x_i^2} \leq \frac{\frac{\partial g_{2i}(w_i^*, x_0)}{\partial x_i}}{\frac{\partial g_{1i}(w_i^*, x_0)}{\partial x_i}} [\frac{\partial^2 g_{1i}(w_i^*, x_0)}{\partial x_i^2}]$$

These results are obtained by applying directly the diffidence theorem to state dependent preferences. Note that the function $g_{ji}(w_i^*, x_i)$ can represent either a utility function or a marginal utility function (see Gollier and Kimball (1994) for details).

3.1 Risk aversion

By definition of risk aversion, for all actuarial lotteries, we have:

$$\forall (w_s^*, w_t^*), \tilde{x}_i : p_s x_s + p_t x_t = 0 \Rightarrow p_s U_s(w_s^* + x_s) + p_t U_t(w_t^* + x_t) < p_s U_s(w_s^*) + p_t U_t(w_t^*)$$

[2]According to Gollier and Kimball (1996), one motivation for the diffidence theorem with state independent preferences is the presence of more than one random variable. See Pratt and Zeckhauser (1987) for an analysis of such situation. Here we consider a different motivation.

[3]With the hypothesis that g_{1i} and g_{2i} are twice differentiables with respect to x_i. When x_i is evaluated at x_0, we assume that $[\frac{\partial g_{1i}(w_i^*, x_0)}{\partial x_i}] = [\frac{\partial g_{1i}(w_i^*, x_i)}{\partial x_i}]|_{x_i = x_0} \neq 0$ and $[\frac{\partial g_{2i}(w_i^*, x_0)}{\partial x_i}] = [\frac{\partial g_{2i}(w_i^*, x_i)}{\partial x_i}]|_{x_i = x_0}$ exists.

Proposition 1 *Let assume that:*

$$NC1 \qquad \forall {w_i}^* : U_i^{'}(w_i^*) > 0$$
$$NC2 \qquad \forall {w_i}^* : U_i^{''}(w_i^*) < 0$$

Then, from the diffidence theorem, the following condition is necessary and sufficient to obtain risk aversion $\forall i \in S$:

$$\forall {w_i}^*, x_i : U_i(w_i^* + x_i) - U_i(w_i^*) < U_i^{'}(w_i^*)x_i$$

Proof: By a direct application of the diffidence theorem to state dependent preferences.

The above necessary and sufficient condition is equivalent to

$$\frac{x_s^2}{2}U_s^{''}(w_s^*) < 0 \qquad and \qquad \frac{x_t^2}{2}U_t^{''}(w_t^*) < 0$$

i.e. that to obtain risk aversion, the utility function must be strictly concave in each state of nature. We now reinterpret the necessary and sufficient condition in terms of the reference set. According to relation (6) in the reference set and to the fact that $U_i(w_i)$ has to be strictly concave for all $i \in S$, Proposition (1) implies that $f_t^{'}(w_s^*) > 0$ i.e. that the reference set has to be strictly increasing in wealth.

3.2 Prudence

The Prudence concept can be derived from the following relationship:

$$\forall (w_s^*, w_t^*), \tilde{x}_i : p_s x_s + p_t x_t = 0 \Rightarrow p_s U_s^{'}(w_s^* + x_s) + p_t U_t^{'}(w_t^* + x_t) > p_s U_s^{'}(w_s^*) + p_t U_t^{'}(w_t^*).$$

Proposition 2 *Let assume that:*

$$NC1 \qquad \forall {w_i}^* : -U_i^{''}(w_i^*) > 0$$
$$NC2 \qquad \forall {w_i}^* : U_i^{'''}(w_i^*) > 0$$

Then, from the diffidence theorem, the following condition is necessary and sufficient to obtain prudence $\forall i \in S$:

$$\forall {w_i}^*, x_i : U_i^{'}(w_i^* + x_i) - U_i^{'}(w_i^*) > U_i^{''}(w_i^*)x_i$$

Proof: By a direct application of the diffidence theorem.

Proposition 2 implies a further restriction on the reference set. Indeed from Proposition 2, we have that:

$$U_s^{'''}(w_s^*) > 0 \qquad and \qquad U_t^{'''}(w_t^*) > 0.$$

Using (7) in the reference set, we must then have that:

$$U_t^{'''}(w_t^*) + U_t^{''}(w_t^*)\frac{f_t^{''}(w_s^*)}{f_t^{'2}(w_s^*)} > 0 \Rightarrow U_t^{'''}(w_t^*) > 0$$

and

$$\frac{U_s^{'''}(w_s^*)}{f_t^{'2}(w_s^*)} - U_t^{''}(w_t^*)\frac{f_t^{''}(w_s^*)}{f_t^{'}(w_s^*)} > 0 \Rightarrow U_s^{'''}(w_s^*) > 0.$$

So, by using (9) and the above relationships, this means that the following condition on the reference set

$$-P_t(w_t^*)f_t^{'}(w_s^*) < \frac{f_t^{''}(w_s^*)}{f_t^{'}(w_s^*)} < P_s(w_s^*)$$

is necessary and sufficient to obtain prudence for a risk averse individual.

Consequently the reference set has not to be linear to obtain prudence. Moreover, it can be either strictly concave or strictly convex. Another interesting feature is that the reference set function has not to be unique in the sense that the above condition can hold for any linear and increasing transformation of $f_t(w_s^*)$ or $U_i(w_i^*)$.

3.3 Decreasing risk aversion

Consider now the decreasing risk aversion property which implies that $U_i^{''}(w_i) < 0$ and $U_i^{'''}(w_i) > 0 \, \forall i \in S$ or that

$$\forall (w_s^*, w_t^*), \tilde{x}_i : p_s U_s(w_s^* + x_s) + p_t U_t(w_t^* + x_t) < p_s U_s(w_s^*) + p_t U_t(w_t^*)$$
$$\Rightarrow p_s U_s^{'}(w_s^* + x_s) + p_t U_t^{'}(w_t^* + x_t) > p_s U_s^{'}(w_s^*) + p_t U_t^{'}(w_t^*).$$

Proposition 3 *Assume that:*

$$NC1 \qquad \forall {w_i}^* : A_i(w_i^*) > 0$$
$$NC2 \qquad \forall {w_i}^* : P_i(w_i^*) > A_i(w_i^*)$$

then from the diffidence theorem, the following condition is necessary and sufficient for decreasing absolute risk aversion $\forall i \in S$:

$$\forall {w_i}^*, x_i : -U_i^{'}(w_i^* + x_i) + U_i^{'}(w_i^*) < A_i(w_i^*)[U_i(w_i^* + x_i) - U_i(w_i^*)]$$

Proof: By a direct application of the diffidence theorem.

These conditions are equivalent to[4]:

$$P_s(w_s^*) > A_s(w_s^*) \qquad and \qquad P_t(w_t^*) > A_t(w_t^*).$$

If now we look at the reference set conditions, by using relations (8) and (9) in the reference set, we must have

$$P_t(w_t^*)f_t^{'}(w_s^*) - \frac{f_t^{''}(w_s^*)}{f_t^{'}(w_s^*)} > A_t(w_t^*)f_t^{'}(w_s^*) \Rightarrow P_t(w_t^*) > A_t(w_t^*)$$

and

$$\frac{P_s(w_s^*)}{f_t^{'}(w_s^*)} + \frac{f_t^{''}(w_s^*)}{f_t^{'2}(w_s^*)} > \frac{A_s(w_s^*)}{f_t^{'}(w_s^*)} \Rightarrow P_s(w_s^*) > A_s(w_s^*).$$

So, the reference set $f_t(w_s^*)$ for a risk averse individual must be such that

$$-[P_t(w_t^*) - A_t(w_t^*)]f_t^{'}(w_s^*) < \frac{f_t^{''}(w_s^*)}{f_t^{'}(w_s^*)} < [P_s(w_s^*) - A_s(w_s^*)]$$

to obtain decreasing absolute risk aversion.

Again, the reference set has not to be linear to obtain decreasing absolute risk aversion with state dependent preferences. In the Appendix, we present examples of decreasing absolute risk aversion (DARA) utility functions that are compatible with non-linear reference sets. For Karni (1985, p.40), it is not clear whether autocomparability is a necessary condition for DARA. Our contribution clearly shows that this condition is not necessary.

4 Conclusion

An individual is said to be risk averse, if and only if his utility function is strictly concave in each state of nature. This means that the reference set must be strictly

[4]See Dionne and Fombaron (1996) for a similar condition to obtain a convex efficiency frontier under adverse selection.

increasing in wealth. The prudence notion is applicable to a risk averse individual with state dependent preferences, if and only if the latter is prudent in each state of nature, which implies a restrictive necessary and sufficient condition on the reference set. The linearity of the reference set is a sufficient but not a necessary condition to obtain prudence.

For absolute risk aversion to be decreasing in weatlh, prudence must be greater than absolute risk aversion in each state of nature, but, again, the reference set has not to be linear.

References

[1] Dehez, Pierre and Jacques H. Drèze (1982), *State Dependent Utility, the Demand for Insurance and the Value of Safety*, in The Value of Life and Safety, M. Jones-Lee, editor, North Holland, Amsterdam, New York, pp.41-65.

[2] Dionne, Georges (1982), *Moral Hazard and State Dependent Utility Function*, The Journal of Risk and Insurance, Vol.49, pp.405-422.

[3] Dionne, Georges (1990), *Review Article of Drèze's Essays on Economic Decisions under Uncertainty*, The Geneva Papers on Risk and Insurance Theory, Vol.15,No.2, pp.193-202.

[4] Dionne, Georges and Nathalie Fombaron (1996), *Non-convexities and the Efficieny of Equilibria in Insurance Markets with Asymmetric Information*, Economics Letters, Vol.52, pp.31-40.

[5] Drèze, Jacques H. (1986), *Moral Expectation with Moral Hazard*, in Contributions to Mathematical Economics, In Honor of Gérard Debreu, Werner Hildenbrand and Andreu Mas-Colell, editors, North Holland, Amsterdam, pp.187-204.

[6] Drèze, Jacques H. (1987), *Decision Theory with Moral Hazard and State-Dependent Preferences*, in Essays on Economic Decisions Under Uncertainty, Jacques H. Drèze, editor, Cambridge University Press, Cambridge, UK, pp.23-70.

[7] Eeckhoudt, Louis and Miles S. Kimball (1992), *Background Risk, Prudence, and the Demand for Insurance*, in Contributions to Insurance Economics, Georges Dionne, editor, Kluwer Academic Publishers, pp.239-254.

[8] Gollier, Christian and Miles S. Kimball (1994), *Toward a Systematic Approach to the Economic Effects of Uncertainty II: Characterizing Utility Functions*, unpublished manuscript, University of Toulouse.

[9] Gollier, Christian and Miles S. Kimball (1996), *New Methods in the Classical Economics of Uncertainty: Comparing Risks*, Working Paper, University of Toulouse.

[10] Karni, Edi (1983), *Risk Aversion for State Dependent Utility Functions: Measurement and Applications*, International Economic Review, Vol.24, pp.637-647.

[11] Karni, Edi (1985), *Decision Making under Uncertainty, The Case of State-Dependent Preferences*, Havard University Press, Cambridge, Massachusetts and London, England.

[12] Kimball Miles S. (1990), *Precautionary Saving in the Small and in the Large*, Econometrica, Vol.58, pp.53-73.

[13] Pratt, J.W. and R. Zeckhauser (1987), *Proper Risk Aversion*, Econometrica, Vol.55, pp.143-154.

[14] Roëll, Ailsa (1984), *Essays in the Theory of Comparative Risk Aversion; with Special Reference to the Case of State-Dependent Preferences*, Ph.D. Dissertation, John Hopkins University.

[15] Rustichini, Aldo and Jacques H. Drèze (1994), *State Dependent Utility*, Discussion Paper No.9443, CORE.

[16] Shavell, Steven (1978), *Theoretical Issues in Medical Malpractice*, in The Economics of Medical Malpractice, Rottenberg S., editor, American Ent. Inst., Washington.

APPENDIX: EXAMPLES

Example 1: Consider two states of nature $s, t \in S$, with the respective utility functions:

$$\begin{aligned} U_s(w_s) &= w_s^{\frac{1}{2}} & U_t(w_t) &= log(w_t) \\ U_s'(w_s) &= \frac{1}{2} w_s^{-\frac{1}{2}} & U_t'(w_t) &= \frac{1}{w_t} \\ -U_s''(w_s) &= \frac{1}{4} w_s^{-\frac{3}{2}} & -U_t''(w_t) &= \frac{1}{w_t^2} \\ U_s'''(w_s) &= \frac{3}{8} w_s^{-\frac{5}{2}} & U_t'''(w_t) &= \frac{2}{w_t^3} \\ -\frac{U_s''(w_s)}{U_s'(w_s)} = A_s(w_s) &= \frac{1}{2w_s} & -\frac{U_t''(w_t)}{U_t'(w_t)} = A_t(w_t) &= \frac{1}{w_t} \\ -\frac{U_s'''(w_s)}{U_s'(w_s)} = P_s(w_s) &= \frac{3}{2w_s} & -\frac{U_t'''(w_t)}{U_t'(w_t)} = P_t(w_t) &= \frac{2}{w_t} \end{aligned}$$

We can define the reference set:

$$U_s'(w_s^*) = U_t'(w_t^*) \Rightarrow \frac{1}{2}(w_s^*)^{-\frac{1}{2}} = \frac{1}{w_t^*} \Rightarrow w_t^* = 2(w_s^*)^{\frac{1}{2}} = f_t(w_s^*)$$

with

$$f_t'(w_s^*) = (w_s^*)^{-\frac{1}{2}} > 0$$

and

$$f_t''(w_s^*) = -\frac{1}{2(w_s^*)^{\frac{3}{2}}} < 0.$$

The reference set function is strictly concave. The necessary and sufficient condition to obtain prudence becomes

$$-P_s(w_s^*) < \frac{f_t''(w_s^*)}{f_t'(w_s^*)} < P_t(w_t^*) f_t'(w_s^*) \Rightarrow -\frac{3}{2w_s^*} < -\frac{1}{2w_s^*} < \frac{1}{w_s^*}.$$

For decreasing absolute risk aversion, we obtain:

$$-[P_s(w_s^*) - A_s(w_s^*)] < \frac{f_t''(w_s^*)}{f_t'(w_s^*)} < [P_t(w_t^*) - A_t(w_t^*)] f_t'(w_s^*) \Rightarrow -\frac{1}{w_s^*} < -\frac{1}{2w_s^*} < \frac{1}{2w_s^*}.$$

We observe that the linearity of the reference set is not necessary. We must now verify that the measure of absolute risk aversion, or the risk premium, is decreasing.

Let introduce the following lottery $(-1, 1/9) = (x_s, x_t)$ with probabilities $(0.1, 0.9) = (p_s, p_t)$, around the two initial situations $(1, 2) = (w_s^*, w_t^*)$ and $(2, 2^{3/2}) = (w_s^{*'}, w_t^{*'})$ both in the reference set. We obtain $(0.902, 1.900)$ for the certainty equivalent of the final wealths $(0, 19/9)$ corresponding to the initial wealths $(1, 2)$ and yielding the following risk premium: 0.1001. We observe that the risk premium decreases when initial wealths increase at $(2, 2^{3/2})$, having the value 0.0190 at the final wealths $(1, 2^{3/2} + 1/9)$ that correspond to the certainty equivalent $(1.974, 2.810)$.
Other values are given in the next table:

Final wealth	Reference Set	Certainty Equivalent	Risk Premium
$(0, 2.111)$	$(1, 2)$	$(0.902, 1.900)$	0.1001
$(\frac{1}{2}, 2.560)$	$(\frac{3}{2}, 2.449)$	$(1.466, 2.421)$	0.0288
$(1, 2.939)$	$(2, 2.828)$	$(1.974, 2.810)$	0.0190
$(\frac{3}{2}, 3.273)$	$(\frac{5}{2}, 3.162)$	$(2.478, 3.148)$	0.0144
$(2, 3.575)$	$(3, 3.464)$	$(2.981, 3.453)$	0.0116
$(9, 6.436)$	$(10, 6.325)$	$(9.991, 6.322)$	0.0035

Table 1: Variation of the risk premium in the first example.

Example 2: Consider now the following state dependent utility functions:

$$U_s(w_s) = 1 - w_s^{-\frac{1}{2}} \qquad U_t(w_t) = log(w_t)$$
$$U_s'(w_s) = \frac{1}{2} w_s^{-\frac{3}{2}} \qquad U_t'(w_t) = \frac{1}{w_t}$$
$$-U_s''(w_s) = \frac{3}{4} w_s^{-\frac{5}{2}} \qquad -U_t''(w_t) = \frac{1}{w_t^2}$$
$$U_s'''(w_s) = \frac{15}{8} w_s^{-\frac{7}{2}} \qquad U_t'''(w_t) = \frac{2}{w_t^3}$$
$$-\frac{U_s''(w_s)}{U_s'(w_s)} = A_s(w_s) = \frac{3}{2w_s} \qquad -\frac{U_t''(w_t)}{U_t'(w_t)} = A_t(w_t) = \frac{1}{w_t}$$
$$-\frac{U_s'''(w_s)}{U_s'(w_s)} = P_s(w_s) = \frac{5}{2w_s} \qquad -\frac{U_t'''(w_t)}{U_t'(w_t)} = P_t(w_t) = \frac{2}{w_t}$$

The reference set is defined by:

$$U_s'(w_s^*) = U_t'(w_t^*) \Rightarrow \frac{1}{2}(w_s^*)^{-\frac{3}{2}} = \frac{1}{w_t^*} \Rightarrow w_t^* = 2(w_s^*)^{\frac{3}{2}} = f_t(w_s^*)$$

with

$$f_t'(w_s^*) = 3(w_s^*)^{\frac{1}{2}} > 0$$

and

$$f_t''(w_s^*) = \frac{3}{2(w_s^*)^{\frac{1}{2}}} > 0.$$

In this case the reference set function is strictly convex. The necessary and sufficient condition to obtain prudence is

$$-P_s(w_s^*) < \frac{f_t''(w_s^*)}{f_t'(w_s^*)} < P_t(w_t^*)f_t'(w_s^*) \Rightarrow -\frac{5}{2w_s^*} < \frac{1}{2w_s^*} < \frac{3}{w_s^*}.$$

For decreasing absolute risk aversion, we obtain:

$$-[P_s(w_s^*) - A_s(w_s^*)] < \frac{f_t''(w_s^*)}{f_t'(w_s^*)} < [P_t(w_t^*) - A_t(w_t^*)]f_t'(w_s^*) \Rightarrow -\frac{1}{w_s^*} < -\frac{1}{2w_s^*} < \frac{3}{2w_s^*}.$$

Again, we observe that the linearity of the reference set is not necessary to obtain decreasing absolute risk aversion. Also, we can verify that the measure of absolute risk aversion, or the risk premium, is decreasing.

Indeed, by using the same lottery $(-1, 1/9) = (x_s, x_t)$ with the same probabilities $(0.1, 0.9) = (p_s, p_t)$ around the two initial situations $(1.5, 3.67) = (w_s^*, w_t^*)$ and $(2, 5.66) = (w_s^{*'}, w_t^{*'})$ both in the reference set, we have $(1.46, 3.55)$ for the certainty equivalent of the final wealths $(0.5, 3.78)$ corresponding to the initial wealths $(1.5, 3.67)$ and yielding the following risk premium: 0.11896. We observe that the risk premium decreases when initial wealths increase at $(2, 5.66)$, having the value 0.0662 at the final wealths $(1, 5.77)$ that correspond to the certainty equivalent $(1.89, 5.58)$. Other values are given in the next table:

Final wealth	Reference Set	Certainty Equivalent	Risk Premium
$(\frac{1}{2}, 3.785)$	$(\frac{3}{2}, 3.674)$	$(1.465, 3.546)$	0.1190
$(1, 5.768)$	$(2, 5.657)$	$(1.893, 5.585)$	0.0662
$(\frac{3}{2}, 8.017)$	$(\frac{5}{2}, 7.906)$	$(2.490, 7.856)$	0.0460
$(2, 10.403)$	$(3, 10.392)$	$(2.993, 10.354)$	0.0353
$(9, 63.357)$	$(10, 63.246)$	$(9.999, 63.236)$	0.0084

Table 2: Variation of the risk premium in the second example.