Experience Rating Schemes for Fleets of Vehicles

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Abstract

This paper proposes bonus-malus systems for ‡eets of vehicles, by using the individual characteristics of both the vehicles and the carriers. Bonus-malus coe¢cients are computed from the history of claims or from the history of safety o¤ences of the carriers and the drivers. The empirical results are derived from a data set obtained from the Société de l'Assurance Automobile du Québec, the public insurer for bodily injuries and the regulator of road safety.

Keywords: Strati…ed portfolios, credibility, vehicle, ‡eet, accidents, safety o¤ences.

JEL Numbers: D80, C23, C35, G22.

Résumé

Ce cahier propose deux systèmes de bonus-malus utilisant les caractéristiques des véhicules et des transporteurs pour mesurer les risques des ‡ottes de véhicules. Les coe¢cients bonus-malus sont calculés des historiques d'accidents ou des infractions au code de la sécurité routière. Les résultats empiriques sont obtenus de données compilées par la Société de l'Assurance Automobile du Québec, l'assureur public pour les dommages corporels et l'agence de réglementation de la sécurité routière au Québec.

Mots clés : Portefeuille strati…é, crédibilité, véhicule, ‡otte, accidents, infractions au code de la sécurité routière.

Classi…cation JEL: D80, C23, C35, G22.

1 Introduction

This paper stems from a study carried out for the Société de l'Assurance Automobile du Québec, later referred as the SAAQ (see Dionne, Desjardins, Pinquet (1999, 2000)). Its objective is to provide Bonus-Malus Systems (BMS) for ‡eets of vehicles from the history of claims or from that of safety o¤ences.

Fleets of vehicles are owned by …rms, which are commercial motor carriers in the SAAQ portfolio. A portfolio of insurance contracts subscribed by …rms has a strati…ed structure, and the size of the stratum (the set of policies held by a given …rm) is a key variable in risk analysis. The propensity to self-insurance increases with the size of the stratum. Insurance contracts for ‡eets of vehicles often use stop-loss risk sharing schemes (see Marie-Jeanne (1994) for their properties as a function of the ‡eet size, and Teugels, Sundt (1991) for experience rating schemes on the aggregate loss). These rating structures are designed for large ‡eets, which is not the case on average for the portfolio analyzed in this article. Notice that, in general, ‡eet insurance business is o¤ered mostly for ‡eets with little or medium size.

In our data set, the characteristics of each ‡eet are recorded by the SAAQ in real-time (see Section 2.2 for more details), and the tari¤ structures proposed in this article use the individual characteristics of both the vehicles and the carriers. The history of a vehicle should have a greater ability to predict the risk level of this vehicle than that of the other vehicles in the ‡eet. The basic issue in the statistical analysis of the portfolio is the assessment of these predictive abilities. Information on the drivers is not available in the data set, so a new vehicle can only be related to the ‡eet to which it belongs. Bonus-malus coe¢cients for the next period will then depend on an expected turnover for the vehicles of the ‡eet.

The experience rating schemes are based on models with hierarchical random e¤ects (see Jewell (1975)). Two types of bonus-malus systems are analyzed. A bonus-malus system designed from the number of claims is presented in Section 3. The coverage is for bodily injuries. Bonus-malus coe¢cients are obtained from vehicle-speci…c and ‡eet-speci…c credibilities and from an expected turnover for the vehicles of the ‡eet.

Compensations for bodily injuries are performed in Quebec within a pure no-fault framework (Devlin (1992); Boyer and Dionne (1987)), so it is di¢cult to use the history of claims in the rating structure, because standard BMS always have a "crime and punishment" ‡avour. Since 1992, the history of safety o¤ences is used in the tari¤ structure for private vehicles (see Dionne and Vanasse (1997b) for a related study). Experience rating schemes with this approach for ‡eets of vehicles are presented in Section 4.

The BMS designed in Section 3 is consistent with respect to the ‡eet-speci…c components, which is not the case when claims are replaced by safety o¤ences as in Section 4. However, the BMS based on safety o¤ences outperforms the one based on accidents after a year of experience with our data. The explanation of this somewhat surprising …nding is the following. The frequency of o¤ences is fourteen times higher than that of claims with bodily injuries. Even if the BMS based on safety o¤ences is less e¢cient than the one based on accidents in the long run, the former system is closer to its limit in the short run, due to the higher frequency of safety o¤ences.

A short conclusion summarizes the main results and proposes some extensions to the models presented in this article.

2 Economic environment and data set

2.1 Economic environment

Let us precise …rst the context of the study. The Province of Quebec introduced a new Automobile Insurance Act in March, 1978 to govern accident compensation. The Government had two goals in mind in tabling this legislation - to provide a rapid and reliable method for compensating all victims of bodily injuries, and to ensure better control of the cost of car repairs and faster compensation for property damage.

Fault has been entirely eliminated for bodily injuries. Compensation is provided by a compulsory and universal public plan. This plan is administered by a public corporation, the SAAQ. There is a maximum indemnity (which was estimated to compensate the total loss of income of 85 per cent of the population in 1978) for disability and death bene…ts. The indemnities for bodily injury are in lieu of all rights to sue for bodily injuries or death, and no action is admitted before any court of justice.

The pricing procedure is very simple. The main sources of …nancing are from drivers' permits and automobile registration fees. Weight and type of vehicle driven are taken into consideration for vehicles other than pleasure vehicles. Past driving experience is taken into account since 1992 by using demerit points of the drivers.

So the SAAQ is a state insurer which provides motor insurance for bodily injuries in a monopolistic situation. As a state company, the SAAQ is also involved in road safety regulation. Consequently, it has a direct access to the information on individual safety o¤ences. It was decided in 1992 to use such information for the pricing of private cars insurance. Besides their ability of screening risks, experience rating schemes provide incentives to careful driving. Indeed, the frequency of claims decreased by at least …ve per cent since the new regulation (see Dionne and Vanasse (1997) for more details).

The SAAQ also provides insurance for bodily injuries for ‡eets of vehicles. This insurance is also compulsory. Information is brought in real time for each vehicle, a situation which is not often encountered in this market. In order to create road safety incentives, the introduction of an experience rating scheme (as well as an a priori rating structure) is under consideration, which motivated the present study. This type of insurance rating is easy to implement for the SAAQ since it has a direct access to all the necessary data.

2.2 Data set

We created the data bank from the SAAQ …les. Since January 1991, the SAAQ has been mandated to verify that commercial vehicles respect the laws and regulations governing, for example, the vehicle load and size limits, etc. In addition, the SAAQ was also given the mandate to verify the mechanical conformity of the vehicles.

In our working sample, the vehicles were observed during the years 1995 and 1996. The duration of observation of a vehicle is the validity duration of its licence plate. The weight of the vehicles has to be greater than 3000 kgs, hence ‡eets of cars do not belong to this sample. The portfolio contains 50746 ‡eets and 124629 vehicles, and ‡eets are of small size on average. The size of the ‡eet is measured in vehicle-years, which is the sum of the validity durations. The other ‡eet-speci…c rating factors are the age of the …rm and its activity sector. The vehicle-speci…c rating factors are the weight, the type of use, the type of fuel, the number of cylinders and the number of axles.

The initial …le is the …le of all registered motor carriers as of July 23, 1997. To be in that …le a motor carrier must own or lease (long term) one or more eligible vehicles. For administrative reasons, the motor carriers may commit safety violations before being registered in the …le. A vehicle is eligible if

- The vehicle has one of the following license plate category: public, private or school bus, general merchandise transport, bulk transport or commercial vehicles;
- The license plate status is D for detain;
- The type of the vehicle is a bus or a truck;
- The vehicle status is active; and
- The vehicle weight is greater than or equal to 3,000 kgs.

Figure 1 presents the di¤erent steps for the construction of the data set by linking SAAQ …les. The MOTOR CARRIER …le is continuously updated and new information erases previous ones. Over a period of time, a motor carrier may becomes inactive if:

- All his eligible vehicles have no more valid plate;
- He no longer detains a C.T.Q permit; or
- Owner or renter (long term) becomes a short-term rental.

A sequential identi…cation (ID) number, which identi…es a motor carrier, has been extracted from the MOTOR CARRIER …le. This number has been …ltered from CLEDPA …le that links old and new identi…cation numbers. In addition, a motor carrier may be united to other motor carriers, in which case they are written in a …le name FUSION. Another reason for being included in the FUSION …le is for administrative reasons when a motor carrier has more than one identi…cation numbers. In this case, they are paired together. In order to identify motor carriers, we create a resulting identi…cation number from all identi…cation numbers extracted from MOTOR CARRIER, CLEDPA and FUSION …les. This step has been necessary because, in 1996, a carrier could have up to 28 identi…cation numbers.
>From all the identi…cation numbers a motor carrier may have, we matched information concerning all their eligible vehicles written in the REGISTRATION …le, such as characteristics concerning their authorization to circulate written in the AUTHORIZATION …le; characteristics of the vehicles written in the VEHICLE …le, and characteristics of the license plate written in the PLATE …le. Characteristics of safety violations committed by a motor carrier written in the MOTOR CARRIER VIOLATION …le was also matched. From license plate number of all eligible vehicles, crashes and violations committed by drivers have been extracted from ACCIDENT and VIOLATION …les. And, …nally, from the vehicle number, characteristics concerning mechanical conformity of the vehicles has been extracted from the MECHANICAL …le.

Insert Figure 1 about here

All information concerning vehicles and motor carriers were brought together to create a …le for accident analyses. In the models, the unit of observation is a vehicle with at least one day with a valid license plate in 1996 and where dates of creation and dissolution of the motor carrier agreed with those of the authorization to circulate. This yields up to

143,744 vehicles (trucks and buses) associated to 52,662 motor carriers. For this article, we consider only the 124,629 trucks associated to 50,746 motor carriers. In considering the safety offences committed in 1995 in the analyses, 24,581 trucks with no day with a valid license plate in 1995 have been dropped from the data set.

3 Bonus-malus system from the number of claims

3.1 Bonus-malus coefficients as functions of the size of the fleet: Two limit examples

On a stratified portfolio, fixed and random effects introduced to design an optimal BMS must have a hierarchical structure (Jewell (1975)). The risk distribution of each vehicle includes then a vehicle-specific effect and a fleet-specific effect. Let us compute bonus-malus coefficients in two limit situations:

- Only the vehicle-specific effect is retained. The history of a vehicle cannot be used to predict the risk levels of the other vehicles in the fleet. If all the vehicles have the same a priori frequency risk, the credibility computed at the fleet level is the one given to each vehicle. As the variance of the ratio between the number of claims and the frequency premium decreases towards 0 when the size of the fleet goes to infinity, the same result holds for the variance of the bonus-malus coefficient.

- Only the fleet-specific effect is included in the number of claims distribution. Denote m as the number of vehicles in a given fleet, n_i as the number of claims reported by the vehicle i and λ as the a priori frequency risk for all the vehicles. We then have

$$N_i \sim P(\lambda u) \ (\forall i = 1, \ldots, m) \Rightarrow \sum_{i=1}^{m} N_i \sim P(m\lambda u);$$

  if the N_i are independent in the fixed effects model (the fixed effect common to the vehicles in the fleet is denoted as u). If we write $E(U) = 1$; $V(U) = \sigma^2$ in the random effects model, the credibility granted to the fleet in the prediction is equal to

$$\alpha = \frac{m\widehat{\lambda}\widehat{\sigma}^2}{1 + m\widehat{\lambda}\widehat{\sigma}^2}.$$

  This credibility increases towards one when the size m goes to infinity, and the bonus-malus coefficient converges towards the fleet-specific fixed effect u. The variance of the bonus-malus coefficient increases with the size of the fleet in the random effects model.

If the two random effects are included in a hierarchical model, the credibility granted to the history of the fleet will increase with its size if the estimated variance of the fleet-specific random effect is greater than zero. On the other hand, the variance of the bonus-malus coefficients is not a monotonic function of the size of the fleets. The increase of risk revelation with the size of the fleet is balanced by risk compensation between the vehicles.

3.2 Estimation of a model with random effects on a stratified portfolio

The hierarchical nature of the portfolio is taken into account by a double indexation. The fleets are indexed by $f = 1, \ldots, F$, and the vehicles are indexed by $i = 1, \ldots, m_f$, where m_f is the size of the fleet f. If N_{fi} is the number of claims reported by the vehicle i in the fleet f, we write

$$N_{fi} \sim P(\lambda_{fi} u_{fi}), \; f = 1, \ldots, F, \; i = 1, \ldots, m_f$$

in the fixed effects model. The parameter λ_{fi} is a function of rating factors observed at the fleet level or at the vehicle level. The fixed effect u_{fi} represents the residual heterogeneity in the number of claims distribution. We distinguish firm-specific and vehicle specific effects in the regression and heterogeneity components, and write

$$\lambda_{fi} = d_{fi} \exp(x_f \gamma + z_{fi}\delta), \; u_{fi} = r_f s_{fi}. \tag{1}$$

The parameter λ_{fi} is proportional to the duration of observation of the vehicle d_{fi}. The line-vectors x_f and z_{fi} are the regression components connected to the fleet and to the vehicle. The related parameters are represented by the column-vectors γ and δ. The fixed effect u_{fi} splits into a fleet-specific effect r_f and a vehicle-specific effect s_{fi}. Vehicle-specific heterogeneity components could reflect the behaviour of the drivers, if a given vehicle is used by few drivers. Fleet owners may obey (or not) to safety rules related to the mechanical check-up of vehicles, bulk trucking regulation, driving and work hour rules, etc. The financial structure of the carrier (which is not recorded by the SAAQ) probably influences safety activities, and hence the risk level. Economic and empirical results on the relationship between the financial structure of air carriers and safety are given by Dionne et al. (1997).

The preceding distributions hold for real individuals, and the variables $(N_{fi})_{f=1,\ldots,F; i=1,\ldots,m_f}$ are supposed to be independent in the fixed effects model. The random effects $(R_f)_{f=1,\ldots,F}$ and $(S_{fi})_{f=1,\ldots,F; i=1,\ldots,m_f}$ are i.i.d. in each family and mutually independent. If R and S are random variables with these distributions, we suppose that

$$E(R) = E(S) = 1, \; V(R) = V_{RR}, \; V(S) = V_{SS}.$$

The random effects model deals with generic individuals, defined conditionally on the regression components. Within a semiparametric approach, the distributions on the random effects will only be specified by the variances. If $U = RS$, we have

$$E(U) = E(R)E(S) = 1, \; V(U) = V_{UU} = E(R^2)E(S^2) - 1 = V_{RR} + V_{SS} + V_{RR}V_{SS}.$$

With the total variance and covariance formula, we obtain

$$V(N_{fi}) = \lambda_{fi} + \lambda_{fi}^2 V(U_{fi}) = \lambda_{fi} + \lambda_{fi}^2 V_{UU},$$

$$\mathrm{Cov}(N_{fi}, N_{fi'}) = \lambda_{fi}\lambda_{fi'}\mathrm{Cov}(U_{fi}, U_{fi'}) = \lambda_{fi}\lambda_{fi'} V_{RR} \; (i \neq i') \tag{2}$$

in the random effects model. As the size of the portfolio is large, we will use a frequentist approach, and will describe the data by consistent estimators.

Let $\widehat{\lambda}_{fi} = d_{fi}\exp(x_f\widehat{\beta} + z_{fi}\widehat{\delta})$ be the frequency premium computed in the a priori rating model, where $\widehat{\beta}$ and $\widehat{\delta}$ are the maximum likelihood estimators. If data are generated in the random effects model, we have (Pinquet (1999))

$$\widehat{\lambda}_{fi} \to E(N_{fi}) = \lambda_{fi}E(U) = \lambda_{fi}.$$

The expectation is computed in the random effects model. From the moments computed in (2), we obtain the following limits

$$\widehat{V_{RR}} = \frac{\sum_f \sum_{1\le i,i'\le m_f;\, i\neq i'} (n_{fi}-\widehat{\lambda}_{fi})(n_{fi'}-\widehat{\lambda}_{fi'})}{\sum_f \sum_{1\le i,i'\le m_f;\, i\neq i'} \widehat{\lambda}_{fi}\widehat{\lambda}_{fi'}} \to V_{RR},$$

$$\widehat{V_{UU}} = \frac{\sum_{f,i}\left[(n_{fi}-\widehat{\lambda}_{fi})^2 - n_{fi}\right]}{\sum_{f,i}\widehat{\lambda}_{fi}^{\,2}} \to V_{UU}. \tag{3}$$

Thus consistent estimators of $V(U)$ and $V(R)$ are obtained from the estimators derived in the a priori model. Since $V_{UU} = V_{RR} + V_{SS} + V_{RR}V_{SS}$,

$$\widehat{V_{SS}} = \frac{\widehat{V_{UU}} - \widehat{V_{RR}}}{1+\widehat{V_{RR}}}$$

is a consistent estimator of V_{SS}.

Let us interpret these results. The estimator $\widehat{V_{RR}}$ assesses observed contagion between the claims histories connected to different vehicles within the same fleet. If $\widehat{V_{RR}}$ is greater than zero, the positive observed contagion means that the history of a vehicle can reveal hidden features in the risk distributions of every vehicle in the same fleet. The numerator of the ratio which defines the estimator $\widehat{V_{RR}}$ is easily derived from

$$\sum_f \sum_{1\le i,i'\le m_f;\, i\neq i'} (n_{fi}-\widehat{\lambda}_{fi})(n_{fi'}-\widehat{\lambda}_{fi'}) = \sum_f (n_f-\widehat{\lambda}_f)^2 - \sum_{f,i}(n_{fi}-\widehat{\lambda}_{fi})^2,$$

if we write $n_f = \sum_{1\le i\le m_f} n_{fi}$, $\widehat{\lambda}_f = \sum_{1\le i\le m_f}\widehat{\lambda}_{fi}$. We then have

$$\widehat{V_{SS}} > 0 \iff \widehat{V_{UU}} > \widehat{V_{RR}} \tag{4}$$

$$\iff \frac{\sum_{f,i}\left[(n_{fi}-\widehat{\lambda}_{fi})^2 - n_{fi}\right]}{\sum_{f,i}\widehat{\lambda}_{fi}^{\,2}} > \frac{\sum_f \sum_{1\le i,i'\le m_f;\, i\neq i'} (n_{fi}-\widehat{\lambda}_{fi})(n_{fi'}-\widehat{\lambda}_{fi'})}{\sum_f \sum_{1\le i,i'\le m_f;\, i\neq i'} \widehat{\lambda}_{fi}\widehat{\lambda}_{fi'}}$$

$$\Leftrightarrow \frac{\sum_{f,i}\left[(n_{fi}-\hat{\lambda}_{fi})^2 - n_{fi}\right]}{\sum_{f,i}\hat{\lambda}_{fi}^2} > \frac{\sum_{f}\left[(n_{f}-\hat{\lambda}_{f})^2 - n_{f}\right]}{\sum_{f}\hat{\lambda}_{f}^2}.$$

The estimated variance of the vehicle-specific random effect is greater than zero if the relative overdispersion derived at the vehicle level is greater than its counterpart computed at the fleet level.

3.3 Linear credibility predictors

In this section, we compute linear credibility predictors for each vehicle. They are derived from the history of claims observed at the fleet level, whereas the credibility coefficient depends on the vehicle. Let i_0 be a vehicle which belongs to the fleet f_0. The portfolio is observed during one period, and a bonus-malus coefficient is computed for the next one. In order to allow for a turnover in the portfolio, this vehicle may appear at the second period. Predictors are obtained separately for each fleet, and the fleet index is suppressed in order to simplify the notations. The fleet is supposed to contain m vehicles during the first period.

The bonus-malus coefficient for the vehicle i_0 is supposed to depend only on the number of claims reported on the whole fleet. It is written as $\hat{a}_{i_0} + \hat{b}_{i_0}\left(\sum_{i=1}^m n_i\right)$; with

$$(\hat{a}_{i_0}, \hat{b}_{i_0}) = \arg\min_{a,b} \hat{E}\left[\left(U_{i_0} - a - b\left(\sum_{i=1}^m N_i\right)\right)^2\right].$$

The estimated expectation is derived in the random effects model. Notice that no specific weight is given to the history of the vehicle. As $E(U_{i_0}) = 1$, we have

$$\hat{a}_{i_0} + \hat{b}_{i_0}\left(\sum_{i=1}^m n_i\right) = 1 + \hat{b}_{i_0}\left(\sum_{i=1}^m \left(n_i - \hat{\lambda}_i\right)\right) = (1 - \mathrm{cred}_{i_0}) + \mathrm{cred}_{i_0}\frac{\sum_{i=1}^m n_i}{\sum_{i=1}^m \hat{\lambda}_i},$$

with

$$\mathrm{cred}_{i_0} = \hat{b}_{i_0}\left(\sum_{i=1}^m \hat{\lambda}_i\right) = \frac{\widehat{Cov}\left(U_{i_0}, \sum_{i=1}^m N_i\right)}{\hat{V}\left(\sum_{i=1}^m N_i\right)}\left(\sum_{i=1}^m \hat{\lambda}_i\right).$$

Consistent estimators for the individual moments are

$$\widehat{Cov}(U_{i_0}, N_i) = \hat{\lambda}_i \widehat{Cov}(U_{i_0}, U_i) = \begin{cases} \hat{\lambda}_i \hat{V}_R & (i_0 \neq i) \\ \hat{\lambda}_{i_0} \hat{V}_U & (i_0 = i) \end{cases};$$

$$\hat{V}(N_i) = \hat{\lambda}_i + \hat{\lambda}_i^2 \hat{V}_U; \quad \widehat{Cov}(N_i, N_{i'}) = \hat{\lambda}_i \hat{\lambda}_{i'} \hat{V}_R \quad (i \neq i'); \tag{5}$$

with the estimators obtained in the preceding section. In the computation of the credibility coefficient, two situations may happen:

- Either the vehicle was not observed during the first period ($i_0 \neq i \ \forall i = 1, \ldots, m$). From the estimations obtained in (5), we have

$$\mathrm{cred}_{i_0} = \alpha = \frac{\widehat{V_R}\left(\sum_{i=1}^{m} \hat{\lambda}_i\right)}{1 + \left(\widehat{V_R}\left(\sum_{i=1}^{m} \hat{\lambda}_i\right)\right) + \left(\widehat{V_U} - \widehat{V_R}\right)\left(\frac{\sum_{i=1}^{m} \hat{\lambda}_i^2}{\sum_{i=1}^{m} \hat{\lambda}_i}\right)}.$$

This fleet-specific credibility coefficient roughly increases with the estimated variance of the fleet-specific random effect and with the frequency-premium computed at the fleet level.

- Or the vehicle was observed during the first period ($1 \leq i_0 \leq m$). Then

$$\mathrm{cred}_{i_0} = \alpha + \beta_{i_0}; \quad \beta_{i_0} = \frac{\left(\widehat{V_U} - \widehat{V_R}\right)\hat{\lambda}_{i_0}}{1 + \left(\widehat{V_R}\left(\sum_{i=1}^{m} \hat{\lambda}_i\right)\right) + \left(\widehat{V_U} - \widehat{V_R}\right)\left(\frac{\sum_{i=1}^{m} \hat{\lambda}_i^2}{\sum_{i=1}^{m} \hat{\lambda}_i}\right)}. \qquad (6)$$

The credibility coefficient is the sum of the fleet-specific coefficient and of a vehicle-specific coefficient. It can be computed only if the estimated variance of the vehicle-specific effect $\widehat{V_S}$ is greater than zero (which amounts to $\widehat{V_U} > \widehat{V_R}$ from (4)), a condition fulfilled in our data.

Fleets are open in most cases, which means that an endorsement is not brought to the insurance policy after each arrival or departure of a vehicle in the fleet. In this context, bonus-malus coefficients computed at the vehicle level may appear unrealistic. If ρ is the expected turnover for the vehicles of the fleet, a credibility equal to $\alpha + ((1 - \rho)\bar{\beta})$ can be retained at the fleet level, where $\bar{\beta}$ is the average of the β_i.

3.4 Empirical results

Table 1 presents the results of a Poisson model which explains the number of claims reported in 1996 by regression components derived from the rating factors discussed above. The only continuous rating factor is the age of the firm. We observe that the frequency of claims decreases - ceteris paribus - by 3.4% with a supplementary year of age. The other rating factors have a finite number of categories.

In Table 1, the vehicles are weighted by the risk exposure measured by the number of days the vehicle is authorized to circulate. The estimated exponential of the coefficients (written in a multiplicative way) related to the different levels of each rating factor are averaged to one (column ST. COFF., for standardized coefficient). Two advantages are obtained.

- The coefficients do not depend on the category that must be omitted in the regression for each rating factor in order to avoid colinearity.

- These coefficients can be compared to the relative frequency of each category, which is the frequency of claims for one category divided by the global frequency, column REL. FRE. in Table 1. Consider for instance the category "bulk transport" of the rating factor "firm's activity sector". The relative frequency is 1.617, whereas the standardized coe¢cient derived from the Poisson model equals 1.146. From the likelihood equations of the Poisson model, the number of claims equals the sum of the frequency premiums for each level. The ratio 1.617/1.146=1.411 means that the vehicles belonging to this type of ‡eet have, with respect to other rating factors, a frequency risk level which is 41% higher than the average.

Table 1 also provides levels of signi…cance for the coe¢cients estimated in the regression. The P-VALUE column is obtained from a studentized statistic (i.e. the ratio between the estimated coe¢cient and its estimated standard deviation). For each rating factor, the reference group is related to the level which was suppressed in order to avoid colinearity.

TABLE 1

RATING SCORE FOR THE FREQUENCY OF CLAIMS WITH BODILY INJURIES

VARIABLE: FIRM'S ACTIVITY SECTOR

	WEIGHT (%)	REL.FRE.	ST.COFF.	P-VALUE
general merchandise transport	13.7	1.508	1.233	0.011
bulk transport	10.9	1.617	1.146	0.079
short term rental	2.5	0.959	0.840	0.501
independent trucker, other sector	72.9	0.813	0.940	ref. group

VARIABLE: VEHICLES-YEARS

	WEIGHT (%)	REL.FRE.	ST.COFF.	P-VALUE
0 or 1 vehicle-year	31.8	0.758	0.803	ref. group
2 vehicle-years	11.9	0.887	0.920	0.145
3 vehicle-years	7.2	1.032	1.055	0.010
4 to 9 vehicle-years	17.1	1.111	1.083	<0.001
10 to 20 vehicle-years	9.6	1.292	1.177	<0.001
more than 20 vehicle-years	22.4	1.183	1.164	<0.001

VARIABLE: TYPE OF FUEL

	WEIGHT (%)	REL.FRE.	ST.COFF.	P-VALUE
gasoline	20.4	0.430	0.597	<0.001
fuel oil	79.6	1.147	1.104	ref. group

VARIABLE: WEIGHT OF THE VEHICLE

	WEIGHT (%)	REL.FRE.	ST.COFF.	P-VALUE
from 3000 to 3870 kgs	20	0.624	0.718	0.014
from 3871 to 6220 kgs	20	0.674	0.888	0.025
from 6221 to 7620 kgs	20	1.174	1.108	0.982
from 7621 to 8850 kgs	20	1.428	1.174	0.479
more than 8850 kgs	20	1.099	1.110	ref. group

VARIABLE: TYPE OF USE

	WEIGHT (%)	REL.FRE.	ST.COFF.	P-VALUE
commercial use	75.8	0.809	0.969	0.508
bulk transport	10.4	1.724	1.351	0.005
other types of transport	13.8	1.501	0.904	ref. group

VARIABLE: NUMBER OF AXLES

	WEIGHT (%)	REL.FRE.	ST.COFF.	P-VALUE
unknown	1.3	5.706	6.835	<0.001
2 axles, less than 4000 kgs	21.2	0.573	1.174	ref. group
2 axles, more than 4000 kgs	26.9	0.694	0.797	0.023
3 axles	18.0	0.917	0.781	0.022
4 axles	5.4	0.908	0.760	0.028
5 axles	8.8	0.876	0.635	0.001
6 axles and more	18.4	1.775	1.141	0.869

VARIABLE: NUMBER OF CYLINDERS

	WEIGHT (%)	REL.FRE.	ST.COFF.	P-VALUE
1 to 5 cylinders	1.4	0.840	0.982	0.410
6 to 7 cylinders	59.9	1.261	1.122	<0.001
8 cylinders and more	38.7	0.600	0.812	ref. group
Number of vehicles		124,629		

The frequency of claims increases with the size of the fleet. This result could be explained by a greater exposure to risk (as measured by annual mileage) for the vehicles belonging to large fleets. The same reason probably also explains why gasoline-powered vehicles are much less risky than fuel-powered ones.
If annual mileage was not observed for all vehicles, it was estimated for those which had a recent mechanical check-up (54,699 vehicles). The estimation of the rating model with this supplementary variable leads to the following results, with a level of significance equal to 10%.

- The fuel effect disappears.
- The size effect decreases, but remains significant.
- The firm activity sectors are not significant.
- The number of cylinders effect disappears.

Detailed results can be obtained in Desjardins, Dionne, Pinquet (1999).

On the sample, the estimators given in the preceding section are equal to

$$\widehat{V}_{RR} = \frac{\sum_{f}\sum_{1 \le i,i' \le m_f;\, i \ne i'} (n_{fi} - \widehat{\lambda}_{fi})(n_{fi'} - \widehat{\lambda}_{fi'})}{\sum_{f}\sum_{1 \le i,i' \le m_f;\, i \ne i'} \widehat{\lambda}_{fi}\widehat{\lambda}_{fi'}} = 0.153;$$

$$\widehat{V}_{UU} = \frac{\sum_{f,i}\left[(n_{fi} - \widehat{\lambda}_{fi})^2 - n_{fi}\right]}{\sum_{f,i} \widehat{\lambda}_{fi}^2} = 1.121. \qquad (7)$$

The variance of the vehicle-specific random effect is important. This suggests that most of the vehicles are used by few drivers. The history of a vehicle will have much more ability to predict the risk level of this vehicle than that of the other vehicles in the fleet. Bonus-malus coefficients are computed at the fleet level in Table 2, for the two limit values of the turnover. Credibilities of the histories and standard deviations of the bonus-malus coefficients are given for each size level retained in Table 1.

TABLE 2

AVERAGE CREDIBILITIES FOR FLEETS AND VEHICLES

STANDARD DEVIATIONS OF BONUS-MALUS COEFFICIENTS AT THE FLEET LEVEL

Fleet size	α	$\alpha+\beta$	$\sigma_{bonmal_{\alpha}}$ (turnover=100%)	$\sigma_{bonmal_{\alpha+\beta}}$ (turnover=0%)
0 or 1 vehicle-year	0.003	0.019	0.020	0.136
2 vehicle-years	0.006	0.026	0.030	0.126
3 vehicle-years	0.009	0.030	0.037	0.122
from 4 to 9 vehicle-years	0.019	0.041	0.053	0.116
from 10 to 20 vehicle-years	0.048	0.072	0.083	0.129
more than 20 vehicle-years	0.245	0.262	0.189	0.203

The standard deviations of the bonus-malus coefficients are related to a between fleets dispersion of these coefficients. All the averages computed in Table 2 are weighted by the frequency premiums of the fleets. Due to the important value of the variance of the vehicle-specific random effect, the credibility strongly depends on the turnover for fleets with little or medium size. The same result holds for the dispersion of the bonus-malus coefficients. As expected from the conclusion of Section 3.1, the standard deviation of the bonus-malus coefficients is not a monotonic function of the size of the fleet when the turnover is equal to zero.

3.5 Experience rating with gamma distributions for the random effects (expected value principle)

Bonus-malus coefficients obtained at the vehicle level from the approach retained in the preceding section have a very low within fleets dispersion. This is due to the fact that the vehicle only influences these coefficients through the credibility given to the history of the fleet. The within fleets dispersion of the bonus-malus coefficients, as measured by the standard deviation, is at most equal to three per cent of the total dispersion for the different size levels.

A prediction approach derived from an expected value principle (Lemaire (1985), Dionne and Vanasse (1989), Pinquet (1997)) does not constrain ex ante the shape of the bonus-malus coefficients. A greater within fleets dispersion of these coefficients can be expected, as confirmed in Table 3 which contains between fleets and total dispersions of bonus-malus coefficients computed at the vehicle level.

TABLE 3

TOTAL AND BETWEEN FLEETS STANDARD DEVIATIONS OF BONUS-MALUS COEFFICIENTS

Fleet size	¾between	¾total
0 or 1 vehicle-year	0.134	0.137
2 vehicle-years	0.122	0.150
3 vehicle-years	0.122	0.158
from 4 to 9 vehicle-years	0.117	0.174
from 10 to 20 vehicle-years	0.127	0.194
more than 20 vehicle-years	0.202	0.309

Bonus-malus coe¢cients were obtained with random e¤ects drawn from gamma distributions, with the estimators obtained in (7). The maximum likelihood estimators of the parameters of the Poisson model are consistent estimators in the model with random e¤ects (Gouriéroux et al. (1984)). The negative binomial model with random e¤ects (Hausman et al. (1984)) provides a speci…cation which is close to the preceding one (Pinquet (1999)). The turnover of the vehicles was supposed equal to zero in the computations. The between ‡eets dispersions of the bonus-malus coe¢cients are very close to those obtained in Table 2 for the same value of the turnover. This means that using only the history of the ‡eet in the prediction did not entail a loss of e¢ciency for bonus-malus coe¢cients computed at the ‡eet level.

4 Bonus-malus systems from the number of safety offences

4.1 Safety o¤ences used as regression components

Owing to the no-fault setting, the history of claims is not used by the SAAQ. Safety o¤ences can be used to perform experience rating. In our data base, safety o¤ences of di¤erent types were recorded at the carrier level and at the driver level. Those which were recorded in 1995 are added here as regression components in the Poisson model estimated in Table 1. Hence the number of claims reported in 1996 is explained by rating factors and by the safety o¤ences recorded the year before. Each estimated coe¢cient related to a given type of safety o¤ence leads to a relative malus, if this coe¢cient is positive. The safety o¤ences which did entail a malus are presented in Table 4.

TABLE 4

RELATIVE MALI DERIVED FROM SAFETY OFFENCES

Type of safety offence recorded in 1995	related to	relative malus (%)	P-value
Exceeding speed limits	vehicles	42	<0.001
Not wearing the seat belt	vehicles	93	<0.001
Not respecting hazardous goods rules	carriers	105	0.008
Excess load	carriers	12	0.089
Not stopping at an agent's signal	vehicles	38	0.091
Not respecting driving hours rules	carriers	72	0.113
Number of vehicles		100,048	

We retained the vehicles with a positive duration of authorization for the licence plate during 1995 and 1996. Other safety offences which were not retained by the model are the following: exceeding size limits, not respecting bulk trucking regulation, not respecting mechanical check-up rules, driving with a sanction, not stopping at a red light. Many of them are significant when we consider all types of road accidents (property damages and bodily injuries). See Dionne et al. (1999) for more details, including regression results related to the rating factors. An optimal bonus-malus system is designed in the next section from a model with random effects on two types of events, namely the claims and the safety offences.

4.2 The model with random effects

Let INF_{fi} be the number of safety offences recorded on the vehicle i belonging to the fleet f. We write

$$INF_{fi} \sim P(\tau_{fi} t_{fi});$$

where $\tau_{fi} = d_{fi} \exp(x_f \zeta + z_{fi} \eta)$ is the component of $E(INF_{fi})$ which is explained by the duration of exposure to safety offences and by both fleet-specific and vehicle-specific regression components, and where t_{fi} is the fixed effect. The hierarchical structure of the portfolio is taken into account by writing $t_{fi} = p_f q_{fi}$; where p_f and q_{fi} are the fleet-specific and vehicle-specific fixed effects. All the number variables are supposed independent in the fixed effects model. Let $U; R; T$ and P be random variables with the same joint distribution that any random vector such as $(U_{fi}; R_f; T_{fi}; P_f)$ (we use the notations of Section 3.2). The assumption $E(U) = 1$ made in Section 3.2 is relaxed now, because explicit and joint distributions for the random effects lead to log-normal distributions, and the expectation depends then on the variance. Let $\hat{\tau}_{fi} = d_{fi} \exp(x_f \hat{\zeta} + z_{fi} \hat{\eta})$ be the estimation of $E(INF_{fi})$ computed in the Poisson model without fixed or random effects. If data are generated in the random effects model, we have

$$\hat{\lambda}_{fi} \longrightarrow E(N_{fi}) = \lambda_{fi} E(U); \quad \hat{\tau}_{fi} \longrightarrow E(INF_{fi}) = \tau_{fi} E(T): \tag{8}$$

The expectation is computed in the random effects model. From (8) and results similar to those given in (2), we obtain the following limits

$$\widehat{V}_{UT}^{1} = \frac{\sum_{f,i}\left(N_{fi} - \widehat{\lambda}_{fi}\right)\left(INF_{fi} - \widehat{\tau}_{fi}\right)}{\sum_{f,i}\widehat{\lambda}_{fi}\widehat{\tau}_{fi}} \to \frac{Cov(U,T)}{E(U)E(T)},$$

$$\widehat{V}_{RP}^{1} = \frac{\sum_{f}\sum_{1\le i,i'\le m_f;\, i\ne i'}\left(N_{fi} - \widehat{\lambda}_{fi}\right)\left(INF_{fi'} - \widehat{\tau}_{fi'}\right)}{\sum_{f}\sum_{1\le i,i'\le m_f;\, i\ne i'}\widehat{\lambda}_{fi}\widehat{\tau}_{fi'}} \to \frac{Cov(R,P)}{E(R)E(P)},$$

$$\widehat{V}_{PP}^{1} = \frac{\sum_{f}\sum_{1\le i,i'\le m_f;\, i\ne i'}\left(INF_{fi} - \widehat{\tau}_{fi}\right)\left(INF_{fi'} - \widehat{\tau}_{fi'}\right)}{\sum_{f}\sum_{1\le i,i'\le m_f;\, i\ne i'}\widehat{\tau}_{fi}\widehat{\tau}_{fi'}} \to CV^{2}(P) = \frac{V(P)}{E^{2}(P)},$$

$$\widehat{V}_{TT}^{1} = \frac{\sum_{f,i}\left[\left(INF_{fi} - \widehat{\tau}_{fi}\right)^{2} - \widehat{\tau}_{fi}\right]}{\sum_{f,i}\widehat{\tau}_{fi}^{\,2}} \to CV^{2}(T) = \frac{V(T)}{E^{2}(T)}. \qquad (9)$$

The superscript "1" is used for the preceding estimators because they are obtained at the first step of the Newton-Raphson algorithm of likelihood maximization, where the initial value is the m.l.e. for the a priori rating model. For instance, the estimator $\widehat{V}_{RP}^{1}$ reflects the predictive power that safety offences recorded on a given vehicle have on the risk level of every other vehicle in the same fleet. Not surprisingly, the fleet-specific credibility obtained in the next section will depend on this estimator.

4.3 Linear credibility predictors

An optimal BMS using both claims and safety offences would be more efficient than those designed in the preceding sections (see Pinquet (1998) for a comparison of short-term effects). We now consider the case where claims cannot be used and the frequency of claims is predicted from the history of safety violations only.

Let us compute the bonus-malus coefficient for the frequency of claims reported by the vehicle i_0 belonging to the fleet f_0: The fleet index is suppressed in order to simplify the expressions. The bonus-malus coefficient is written as $\widehat{a}_{i_0} + \widehat{b}_{i_0}\left(\sum_{i=1}^{m} inf_i\right)$; with

$$\left(\widehat{a}_{i_0}, \widehat{b}_{i_0}\right) = \arg\min_{a,b} E\left[\left(\frac{U_{i_0}}{E(U_{i_0})} - a - b\sum_{i=1}^{m} INF_i\right)^{2}\right].$$

>From computations similar to those performed in Section 3.3, we obtain the following bonus-malus coefficient

$$bonmal_{i_0} = 1 + \widehat{b}_{i_0}\left(\sum_{i=1}^{m}\left(inf_i - \widehat{\tau}_i\right)\right); \quad \widehat{b}_{i_0} = \frac{\widehat{Cov}\left(\frac{U_{i_0}}{E(U_{i_0})}, \sum_{i=1}^{m} INF_i\right)}{\widehat{V}\left(\sum_{i=1}^{m} INF_i\right)};$$

with

$$\widehat{Cov}\left(\frac{U_{i_0}}{E(U_{i_0})}, \sum_{i=1}^{m} INF_i\right) = \hat{V}_{RP}^{-1}\left(\sum_{i=1}^{m} \hat{\mu}_i\right) + \left((\hat{V}_{UT}^{-1} - \hat{V}_{RP}^{-1})\,\hat{\tau}_{i_0}\right):$$

The last term must be suppressed if the vehicle i_0 is not observed during the first period. Following the computations of Section 3.3, we obtain then

$$\hat{V}\left(\sum_{i=1}^{m} INF_i\right) = \left(\sum_{i=1}^{m} \hat{\mu}_i\right) + \left(\hat{V}_{RP}^{-1}\left(\sum_{i=1}^{m} \hat{\mu}_i\right)^2\right) + \left[(\hat{V}_{UT}^{-1} - \hat{V}_{RP}^{-1})\left(\sum_{i=1}^{m} \hat{\mu}_i^2\right)\right];$$

$$bonmal_{i_0} = (1 - cred_{i_0}) + cred_{i_0}\frac{\sum_{i=1}^{m} inf_i}{\sum_{i=1}^{m} \hat{\mu}_i};$$

$$cred_{i_0} = \alpha\ (i_0 \notin \{1;\dots;m\});\quad cred_{i_0} = \alpha + \beta_{i_0}\ (i_0 \in \{1;\dots;m\}); \tag{10}$$

$$\alpha = \frac{\hat{V}_{RP}^{-1}\left(\sum_{i=1}^{m} \hat{\mu}_i\right)}{1 + \left(\hat{V}_{RP}^{-1}\left(\sum_{i=1}^{m} \hat{\mu}_i\right)\right) + \left[(\hat{V}_{UT}^{-1} - \hat{V}_{RP}^{-1})\left(\frac{\sum_{i=1}^{m} \hat{\mu}_i^2}{\sum_{i=1}^{m} \hat{\mu}_i}\right)\right]};$$

$$\beta_{i_0} = \frac{(\hat{V}_{UT}^{-1} - \hat{V}_{RP}^{-1})\,\hat{\tau}_{i_0}}{1 + \left(\hat{V}_{RP}^{-1}\left(\sum_{i=1}^{m} \hat{\mu}_i\right)\right) + \left[(\hat{V}_{UT}^{-1} - \hat{V}_{RP}^{-1})\left(\frac{\sum_{i=1}^{m} \hat{\mu}_i^2}{\sum_{i=1}^{m} \hat{\mu}_i}\right)\right]}:$$

The fleet-specific credibility coefficient α increases with $\hat{V}_{RP}^{-1}$; a term related to the covariance between the two fleet-specific random effects. The coefficient β_{i_0} is the vehicle-specific credibility. It makes sense only if $\hat{V}_{UT}^{-1} > \hat{V}_{RP}^{-1}$, a condition fulfilled in our data.

4.4 Empirical results

The frequency of claims with bodily injury reported in 1996 is predicted from the number of safety offences recorded in 1995, and we retained the vehicles with a positive duration of authorization for the licence plate during 1995 and 1996. The detailed results of the regression explaining the number of safety offences recorded in 1995 are presented in Table 5. Let us quote two points.

- The annual frequency of recorded offences is equal to 22.2%. It is much superior to that of the claims with bodily injury liability. This will explain later the better short term performance of the prediction designed in this section.

- The frequency of offences increases with the size of the fleet, but decreases for fleets with more than 20 vehicle-years.

TABLE 5

RATING SCORE FOR THE FREQUENCY OF SAFETY OFFENCES

VARIABLE: FIRM'S ACTIVITY SECTOR

	WEIGHT (%)	REL.FRE.	ST.COFF.	P-VALUE
general merchandise transport	13.3	1.269	1.048	0.013
bulk transport	10.7	2.045	0.997	0.314
short term rental	2.5	1.297	1.742	<0.001
independent trucker, other sector	73.5	0.789	0.967	ref. group

VARIABLE: VEHICLES-YEARS

	WEIGHT (%)	REL.FRE.	ST.COFF.	P-VALUE
0 or 1 vehicle-year	32.7	0.953	1.055	ref. group
2 vehicle-years	11.4	1.022	1.119	0.008
3 vehicle-years	7.1	1.147	1.174	<0.001
4 to 9 vehicle-years	17.4	1.262	1.210	<0.001
10 to 20 vehicle-years	9.7	1.256	1.056	0.725
more than 20 vehicle-years	21.7	0.686	0.606	<0.001

VARIABLE: TYPE OF FUEL

	WEIGHT (%)	REL.FRE.	ST.COFF.	P-VALUE
gasoline	22.2	0.392	0.582	<0.001
fuel oil	77.8	1.174	1.119	ref. group

VARIABLE: WEIGHT OF THE VEHICLE

	WEIGHT (%)	REL.FRE.	ST.COFF.	P-VALUE
from 3000 to 3870 kgs	20.0	0.653	0.991	0.371
from 3871 to 6220 kgs	20.0	0.654	0.939	<0.001
from 6221 to 7620 kgs	20.5	1.112	0.960	<0.001
from 7621 to 8850 kgs	19.2	1.517	1.061	0.473
more than 8850 kgs	20.3	1.082	1.050	ref. group

VARIABLE: TYPE OF USE

	WEIGHT (%)	REL.FRE.	ST.COFF.	P-VALUE
commercial use	76.0	0.776	0.927	0.644
bulk transport	10.2	2.356	1.667	<0.001
other types of transport	13.8	1.234	0.911	ref. group

VARIABLE: NUMBER OF AXLES

	WEIGHT (%)	REL.FRE.	ST.COFF.	P-VALUE
unknown	2.3	0.998	1.119	0.047
2 axles, less than 4000 kgs	20.6	0.651	0.984	ref. group
2 axles, more than 4000 kgs	27.6	0.600	0.751	<0.001
3 axles	18.3	0.802	0.696	<0.001
4 axles	5.7	0.893	0.856	0.017
5 axles	8.2	0.987	0.865	0.044
6 axles and more	17.3	2.305	1.836	<0.001

VARIABLE: NUMBER OF CYLINDERS

	WEIGHT (%)	REL.FRE.	ST.COFF.	P-VALUE
1 to 5 cylinders	1.4	0.714	0.834	0.699
6 to 7 cylinders	59.0	1.294	1.130	<0.001
8 cylinders and more	39.6	0.572	0.812	ref. group

Number of vehicles	100,048

As for the random effects, the numerical values of the estimators are

$$\widehat{V}_{UT}^{\infty} = 0.519;\ \widehat{V}_{PP}^{\infty} = 0.465;\ \widehat{V}_{RP}^{\infty} = 0.141;\ \widehat{V}_{TT}^{\infty} = 1.263.$$

These moment-based estimators can be connected to explicit distributions. If log-normal distributions are retained for the random effects, we can write

$$R = \exp(a_1 N_1);\ U_i = \exp(a_1 N_1 + a_2 N_2^i) \Rightarrow U_i = R S_i; S_i = \exp(a_2 N_2^i).$$

The fleet index is suppressed, and the random variables $N_1; (N_2^i)_{i=1,\ldots,m}$ follow independent standard normal distributions. In the same way, we can write

$$P = \exp(a_3 N_1 + a_4 N_3);\ T_i = \exp(a_3 N_1 + a_4 N_3 + a_5 N_2^i + a_6 N_4^i) \Rightarrow T_i = P Q_i;$$

$$Q_i = \exp(a_5 N_2^i + a_6 N_4^i);$$

with similar assumptions on the random variables $N_3; (N_4^i)_{i=1,\ldots,m}$. It is easily seen that

$$N \sim N(0; I_q) \Rightarrow \frac{Cov(^t aN;\ ^t bN)}{E(^t aN)\,E(^t bN)} = \exp(^t ab) - 1 \quad \forall a, b \in \mathbb{R}^q.$$

The moment-based estimators are then connected with the following values

$$a_1 = 0.381;\ a_2 = 0.828;\ a_3 = 0.346;\ a_4 = 0.512;\ a_5 = 0.346;\ a_6 = 0.562.$$

The predictor computed in this section cannot be consistent with respect to the fleet specific component, since the event for which the frequency is predicted is not retained in the history. When the size of the fleet m converges towards infinity, we have

$$\lim_{m \to +\infty} cred_{i_0} = \frac{\widehat{V}_{RP}^{\infty}}{\widehat{V}_{PP}^{\infty}} = 0.303 \quad \forall i_0.$$

The credibility coefficient $cred_{i_0}$ is defined in (10). As we have the following limit

$$\lim_{m \to +\infty} \frac{\sum_{i=1}^{m} INF_i}{\sum_{i=1}^{m} \hat{\lambda}_i} = \frac{P}{E(P)}$$

in the random effects model, the limit of the bonus-malus coefficient is $\lim_{m \to +\infty} bonmal_{i_0} = \left(1 - \frac{\widehat{V}_{RP}^{1}}{\widehat{V}_{P}^{1}}\right) + \left(\frac{\widehat{V}_{RP}^{1}}{\widehat{V}_{P}^{1}} \frac{P}{E(P)}\right)$. Hence, the bonus-malus system is not consistent (the limit should be $R/E(R)$ for a consistent predictor). The limit is the estimated affine regression of $R/E(R)$ with respect to $P/E(P)$.
Although this bonus-malus system is less efficient in the long run than the one based on the number of claims, it is more efficient after one year, as shown in Table 6.

TABLE 6

AVERAGE CREDIBILITIES FOR FLEETS AND VEHICLES

STANDARD DEVIATIONS OF BONUS-MALUS COEFFICIENTS AT THE FLEET LEVEL

Fleet size	α	$\alpha + \beta$	$\sigma_{bonmal_{\alpha}}$	$\sigma_{bonmal_{\alpha+\beta}}$
0 or 1 vehicle-year	0.027	0.094	0.064	0.216
2 vehicle-years	0.049	0.113	0.087	0.198
3 vehicle-years	0.070	0.132	0.107	0.196
from 4 to 9 vehicle-years	0.114	0.168	0.136	0.197
from 10 to 20 vehicle-years	0.175	0.209	0.186	0.222
more than 20 vehicle-years	0.242	0.252	0.220	0.235

Table 6 is obtained in the same way as Table 2. Standard deviations of bonus-malus coefficients are more important in this table for fleets with little or medium size. This BMS is less efficient in the long run than the one presented in Section 3, but it is closer to its limit, due to the higher frequency of safety offences.

5 Conclusion

The objective of this paper was to propose bonus-malus systems for fleets of vehicles. The models were applied to fleets of trucks, but they could be used for other stratified portfolios if individual information on the insurance contracts was available.
Two systems were presented: one based on past accidents and the other based on past safety offenses. It was shown that the former system is more efficient in the long run, while the second is closer to its limit in the short run, a result explained by the higher frequency of safety offences.
Many extensions of this article can be done. We plan to use information on many periods in order to build up a panel. This panel will be very useful to analyze the stability of the bonus-malus systems over time. It will also permit to verify for how long period the system based on safety offences will dominate the one based on accidents. However such extensions will not be straightforward since we will have to introduce dynamic random effects in order to take into account the serial correlations.

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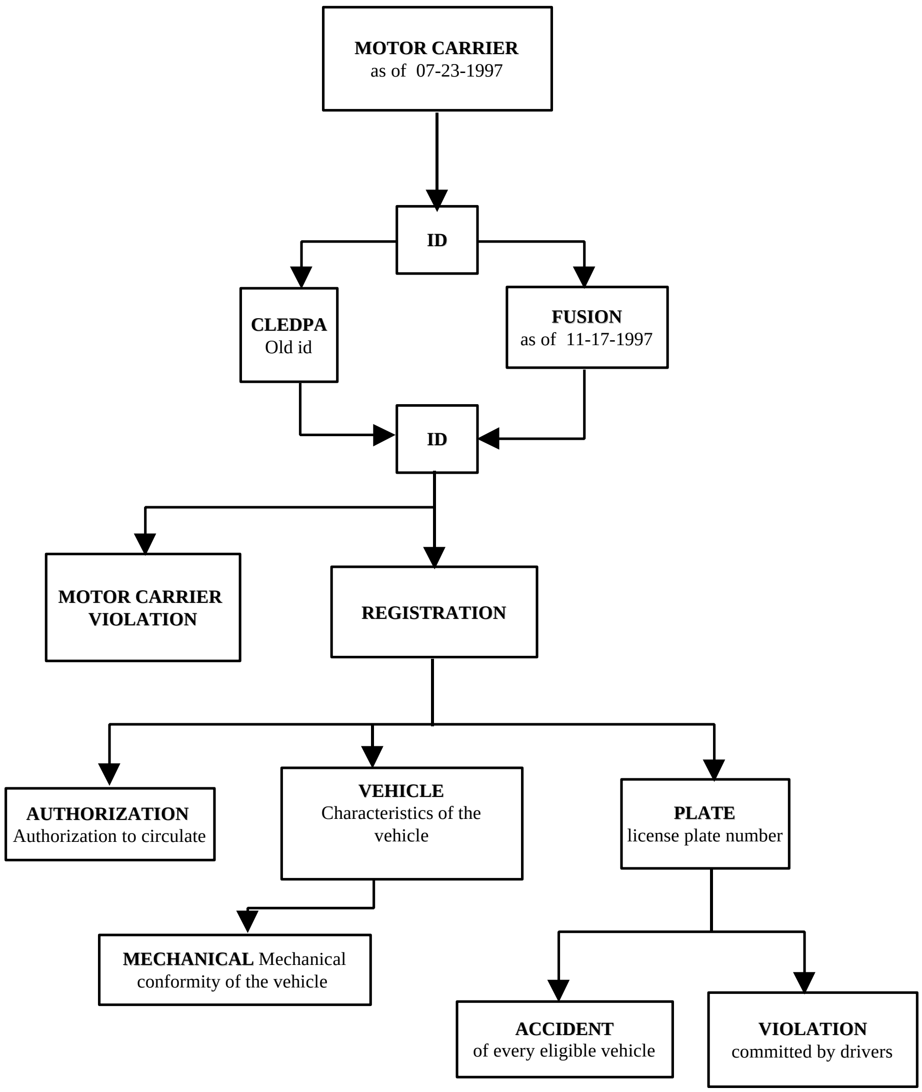

Figure 1: Links between the SAAQ files